1. An Exact Solution of the Equations of Radiative Transfer / C. E. Siewert (Middle East Technical Univ., Turkey), P. F. Zweifel (U of Mich.)

The equations of radiative transfer can, in certain cases, be reduced to equations similar to the equations of neutron transport and they can be solved by techniques similar to those from neutron problems. When the absorption coefficient $K_{\nu}$ for photons of frequency $\nu$ is independent of $\nu$, one speaks of the 'gray case' and it is identical with the one-speed neutron problem. We consider a generalization of the gray case, the 'Uniform Picket Fence Model.' Here $K_{\nu}$ takes two values, $K_1$ and $K_2$. Assuming that the frequency spectrum can be divided into ranges $\Delta \nu_i$ over which the Planck function

$$B_{\nu}(T(z)) = \frac{2h^3}{c^2} [e^{h\nu/kT(z)} - 1]^{-1}$$

is essentially constant, and that in each $\Delta \nu_i$ the fractional range over which $K_{\nu}$ is constant, we obtain equations for the photon energy density similar to the two-group equations. Thus, we have

$$\mu \frac{\partial}{\partial x} \Psi(x, \mu) + \sum \Psi(x, \mu) = C \int_{-1}^{1} \Psi(x, \mu') d\mu'.$$

Here $\Psi(x, \mu)$ has two components $\psi_j(x, \mu)$ defined as the integral of the energy density over the range $\Delta \nu_i$ in which $K_{\nu}$ has the value $K_j$. Also

$$\sum \Psi(x, \mu) = C \int_{-1}^{1} \Psi(x, \mu') d\mu'.$$

Using these results, typical half-range problems can be solved. For the Milne problem, the extrapolated end-point is

$$X_0 = \frac{2\Delta \nu}{\sigma_1 \sigma_2} \left[ \int_{0}^{\sigma_1} \frac{\mu^2}{X(-T)} d\mu + \sigma_2 \int_{0}^{\sigma_1} \frac{\mu^2}{X(-T)} d\mu \right],$$

where $X(z)$ is Case's $X$-function (thought of as a functional of $\Omega(z)$). We also have explicit results for the temperature distribution and the law of darkening (emergent angular distribution) which are too lengthy to be included in this summary.

The case $\tau > 0$ is of great interest in neutron transport. We have succeeded in proving full-range orthogonality and completeness and have solved the infinite-medium Green's function. However, half-space problems, involving half-range completeness, do not appear to be solvable in closed form. We have, though, been able to obtain an explicit solution for the half-space Green's function for $\tau > 0$.

A slightly restricted version of the Milne problem has already been solved by Stewart using another technique. However, our method is simpler; it readily solves other half-space problems (e.g. Albedo problem) and is easily generalized to more than two groups. This generalization is reported elsewhere.

3. STEWART, J. C., unpublished.