DETERMINATION OF THE SINGLE-SCATTERING ALBEDO FROM POLARIZATION MEASUREMENTS OF A RAYLEIGH ATMOSPHERE

(Letter to the Editor)

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Abstract. Exact analysis of a Rayleigh-scattering half space is used to solve the inverse problem whereby the single-scattering albedo can be determined from measurements of the polarized radiation emerging from the atmosphere.

1. Introduction

We consider the equation of transfer

$$\mu \frac{\partial}{\partial \tau} \mathbf{I}(\tau, \mu) + \mathbf{I}(\tau, \mu) = \frac{1}{2} \omega \mathbf{Q}(\mu) \int_{-1}^{1} \mathbf{Q}^T(\mu') \mathbf{I}(\tau, \mu') \, d\mu', \tag{1}$$

applicable to a study of the scattering of polarized light (Chandrasekhar, 1953). Here $\mathbf{I}(\tau, \mu)$ is a vector with components $I_l(\tau, \mu)$ and $I_r(\tau, \mu)$ which are the azimuth-independent angular intensities in the two polarization states. In addition, τ is the optical variable, μ is direction cosine (as measured from the positive τ axis) of the propagating radiation, ω is the single-scattering albedo and

$$\mathbf{Q}(\mu) = \frac{3^{1/2}}{2} \begin{bmatrix} \mu^2 & 2^{1/2}(1 - \mu^2) \\ 1 & 0 \end{bmatrix}. \tag{2}$$

For a typical half-space problem, we generally consider ω to be given, that $\mathbf{I}(\tau, \mu)$ is bounded as τ tends to infinity, that

$$\mathbf{I}(0, \mu) = \mathbf{F}(\mu), \qquad \mu > 0, \tag{3}$$

where $\mathbf{F}(\mu)$ is presumed known, and we seek $\mathbf{I}(\tau, \mu)$ or at least the emerging radiation $\mathbf{I}(0, -\mu)$, $\mu > 0$.

For the inverse problem considered here, we assume that $\mathbf{F}(\mu)$ is given, that we can measure $\mathbf{I}(0, -\mu)$, $\mu > 0$, and thus we seek to determine the single-scattering albedo ω.

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2. Analysis

First of all, we note from a paper by Bond and Siewert (1971) that the exit distribution can be determined from the entering distribution

$$\mathbf{I}(0, -\mu) = \frac{\omega}{2}\mathbf{Q}(\mu)\mathbf{H}(\mu)\int_0^1 \mathbf{H}^T(x)\mathbf{Q}^T(x)\mathbf{F}(x)x\frac{\mathrm{d}x}{x+\mu}, \qquad \mu \in [0, 1], \tag{4}$$

where the $\mathbf{H}$ matrix satisfies the equation

$$\mathbf{H}(\mu) = \mathbf{I} + \mu\mathbf{H}(\mu)\int_0^1 \mathbf{H}^T(x)\mathbf{\Psi}(x)\frac{\mathrm{d}x}{x+\mu}, \qquad \mu \in [0, 1], \tag{5}$$

with

$$\mathbf{\Psi}(x) = \tfrac{1}{2}\omega\mathbf{Q}^T(x)\mathbf{Q}(x). \tag{6}$$

If we now consider

$$S = \int_0^1 \mathbf{F}^T(\mu)\mathbf{I}(0, -\mu)\,\mathrm{d}\mu, \tag{7}$$

then we can use Equation (4) to deduce that

$$S = \frac{\omega}{4}\mathbf{A}\mathbf{A}^T, \tag{8}$$

where

$$\mathbf{A} = \int_0^1 \mathbf{F}^T(\mu)\mathbf{Q}(\mu)\mathbf{H}(\mu)\,\mathrm{d}\mu. \tag{9}$$

If we now consider

$$\mathbf{W} = \int_0^1 \mathbf{Q}^T(\mu)\mathbf{I}(0, -\mu)\,\mathrm{d}\mu, \tag{10}$$

then we can use Equations (4) and (5) to obtain

$$\mathbf{W} = \mathbf{A}^T - \int_0^1 \mathbf{Q}^T(x)\mathbf{F}(x)\,\mathrm{d}x; \tag{11}$$

which can be used in Equation (8) to yield our final result

$$\omega = 4S[\mathbf{P}\mathbf{P}^T]^{-1}, \tag{12}$$

where

$$\mathbf{P} = \int_{-1}^1 \mathbf{Q}^T(\mu)\mathbf{I}(0, \mu)\,\mathrm{d}\mu. \tag{13}$$

Thus when $\mathbf{F}(\mu)$ is given and $\mathbf{I}(0, -\mu)$ is determined experimentally, then S and $\mathbf{P}$ can be computed from Equations (7) and (13), and thus ω is available from Equation (12).

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