DETERMINATION OF THE SINGLE-SCATTERING ALBEDO
FROM POLARIZATION MEASUREMENTS OF A
RAYLEIGH ATMOSPHERE

(Letter to the Editor)

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Abstract. Exact analysis of a Rayleigh-scattering half space is used to solve the inverse problem whereby the single-scattering albedo can be determined from measurements of the polarized radiation emerging from the atmosphere.

1. Introduction

We consider the equation of transfer

\[ \mu \frac{\partial}{\partial \tau} I(\tau, \mu) + I(\tau, \mu) = \frac{1}{2} \omega Q(\mu) \int_{-1}^{1} Q^{\gamma}(\mu') I(\tau, \mu') \, d\mu', \]  

(1)

applicable to a study of the scattering of polarized light (Chandrasekhar, 1953). Here \( I(\tau, \mu) \) is a vector with components \( I_{\parallel}(\tau, \mu) \) and \( I_{\perp}(\tau, \mu) \) which are the azimuth-independent angular intensities in the two polarization states. In addition, \( \tau \) is the optical variable, \( \mu \) is direction cosine (as measured from the positive \( \tau \) axis) of the propagating radiation, \( \omega \) is the single-scattering albedo and

\[ Q(\mu) = \frac{2^{1/2}}{2} \begin{bmatrix} \mu^2 & 2^{1/2}(1 - \mu^2) \\ 1 & 0 \end{bmatrix}. \]  

(2)

For a typical half-space problem, we generally consider \( \omega \) to be given, that \( I(\tau, \mu) \) is bounded as \( \tau \) tends to infinity, that

\[ I(0, \mu) = F(\mu), \quad \mu > 0, \]  

(3)

where \( F(\mu) \) is presumed known, and we seek \( I(\tau, \mu) \) or at least the emerging radiation \( I(0, -\mu), \mu > 0 \).

For the inverse problem considered here, we assume that \( F(\mu) \) is given, that we can measure \( I(0, -\mu), \mu > 0 \), and thus we seek to determine the single-scattering albedo \( \omega \).

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2. Analysis

First of all, we note from a paper by Bond and Siewert (1971) that the exit distribution can be determined from the entering distribution

$$I(0, -\mu) = \frac{\omega}{2} Q(\mu) H(\mu) \int_0^1 H^T(x)Q^T(x)F(x)x \frac{dx}{x + \mu}, \quad \mu \in [0, 1],$$

(4)

where the $H$ matrix satisfies the equation

$$H(\mu) = I + \mu H(\mu) \int_0^1 H^T(x)\Psi(x) \frac{dx}{x + \mu}, \quad \mu \in [0, 1],$$

(5)

with

$$\Psi(x) = \frac{1}{2\omega}Q^T(x)Q(x).$$

(6)

If we now consider

$$S = \int_0^1 F^T(\mu)I(0, -\mu) \ d\mu,$$

(7)

then we can use Equation (4) to deduce that

$$S = \frac{\omega}{4} AA^T,$$

(8)

where

$$A = \int_0^1 F^T(\mu)Q(\mu)H(\mu) \ d\mu.$$ 

(9)

If we now consider

$$W = \int_0^1 Q^T(\mu)I(0, -\mu) \ d\mu,$$

(10)

then we can use Equations (4) and (5) to obtain

$$W = A^T - \int_0^1 Q^T(x)F(x) \ dx;$$

(11)

which can be used in Equation (8) to yield our final result

$$\omega = 4S[PP^T]^{-1},$$

(12)

where

$$P = \int_{-1}^1 Q^T(\mu)I(0, \mu) \ d\mu.$$ 

(13)

Thus when $F(\mu)$ is given and $I(0, -\mu)$ is determined experimentally, then $S$ and $P$ can be computed from Equations (7) and (13), and thus $\omega$ is available from Equation (12).
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References