REMOTE SOUNDING OF AEROSOLS FROM MULTIPLE SCATTERING MEASUREMENTS

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INTRODUCTION

In remote sensing of aerosols, information is generally sought either from transmitted or from once scattered light. But in many cases only the global multiply scattered radiation field can be measured and this makes the inversion problem much more complicated. Weinman et al {1} considered light scattered at small angles in order to derive the forward part of the phase function ; it was approximated by a sum of Gaussian functions. Siewert {2} has reported a concise set of equations that can, in principle, be solved to give the scattering law in terms of the optical thickness of the layer and the radiation field at the two surfaces.
Mc Cormick {3} used moments of the emerging radiance on both sides of the layer to develop equations to be solved for the expansion coefficients of the phase function ; explicit results were given for the highest-order and next-highest-order coefficients. We will follow here another method proposed by Siewert {4}. We assume that the optical thickness is known and we invert radiance measurements to obtain the single-scattering albedo and the phase function.
Then we will show that the equivalent layers built by varying the optical thickness correspond only to different expressions of the same layer. Therefore all the fundamental characteristics of the scattering layer, including the optical thickness, can in principle be uniquely retrieved. However when the radiation field is governed by only the first expansion coefficients, similarity relations can conveniently be used.Finally we discuss the practical application of the method for the clouds of Venus.

RETRIEVAL OF THE PHASE FUNCTION

We consider a plane-parallel homogeneous atmosphere illuminated on the upper boundary by the solar beam. The fundamental characteristics of the scattering layer are the optical thickness τ_1, the single-scattering albedo ω_o, and the phase function $p(\Theta)$, where Θ is the scattering angle. We assume that the optical thickness is known (for example from a measurement of the directly transmitted solar flux) and we seek to invert the measured radiance in order to derive the single-scattering albedo and the phase function.

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The phase function is assumed to be expanded in a series of Legendre polynomials, i.e.,

$$p(\Theta) = \sum_{\ell=0}^{\infty} \beta_\ell \, P_\ell(\cos\Theta) \quad , \tag{1}$$

and the coefficients β_ℓ are sought. The equation of transfer which defines the radiance $I(\tau,\mu,\phi)$ is

$$\mu \frac{\partial}{\partial\tau} I(\tau,\mu,\phi) = I(\tau,\mu,\phi) - \frac{\omega_0}{4\pi} \int_0^{2\pi} \int_{-1}^{1} p(\mu,\phi;\mu',\phi') \, I(\tau,\mu',\phi') \, d\mu' \, d\phi' \quad , \tag{2}$$

where τ is the optical depth measured from the upper boundary, $\cos^{-1} \mu$ is the zenith angle and ϕ the azimuth angle. The boundary conditions are

$$I(o,\mu<o,\phi) = \delta(\mu-\mu_0) \, \delta(\phi-\phi_0) \, \pi F \quad , \tag{3a}$$

and

$$I(\tau_1,\mu>o,\phi) = o \quad , \tag{3b}$$

where δ is the Dirac delta functional. Integrating Eq. (2) over ϕ and using Eq. (1), we find

$$\mu \frac{\partial}{\partial\tau} I(\tau,\mu) = I(\tau,\mu) - \frac{\omega_0}{2} \sum_{\ell=0}^{\infty} \beta_\ell \, P_\ell(\mu) \int_{-1}^{1} P_\ell(\mu') \, I(\tau,\mu') \, d\mu' \quad , \tag{4}$$

where

$$I(\tau,\mu) = \frac{1}{2\pi} \int_0^{2\pi} I(\tau,\mu,\phi) \, d\phi \quad . \tag{5}$$

If we multiply Eq. (4) by $P_\ell(\mu)$ and integrate over μ, we find

$$\ell \frac{d}{d\tau} \psi_{\ell-1}(\tau) + (\ell+1) \frac{d}{d\tau} \psi_{\ell+1}(\tau) = (2\ell+1) \, \Delta_\ell \, \psi_\ell(\tau) \quad , \tag{6}$$

where

$$\psi_\ell(\tau) = \int_{-1}^{1} P_\ell(\mu) \, I(\tau,\mu) \, d\mu \quad , \tag{7}$$

and

$$\Delta_\ell = 1 - \frac{\omega_0 \, \beta_\ell}{2\ell+1} \quad . \tag{8}$$

Note that the net flux is

$$F(\tau) = 2 \, \pi \, \psi_1(\tau) \quad ,$$

and the spherical flux is

$$\Phi(\tau) = 2 \, \pi \, \psi_0(\tau) \quad .$$

Integrating Eq. (6) over τ, we can write

$$(2\ell+1) \, \Delta_\ell \, M_\ell^0 = N_\ell \quad , \tag{9}$$

where

$$M_\ell^\alpha = \int_0^\tau \tau^\alpha \, \psi_\ell(\tau) \, d\tau \quad , \tag{10}$$

and

$$N_\ell = (\ell+1)\,\{\psi_{\ell+1}(\tau_1) - \psi_{\ell+1}(o)\} + \ell\{\psi_{\ell-1}(\tau_1) - \psi_{\ell-1}(o)\} \; . \qquad (11)$$

Equations (8) and (9) express the single-scattering albedo ω_o and the coefficients β_ℓ in terms of the diffuse radiative field. We note that N_ℓ can be computed from the measured outgoing radiance on both sides of the layer. The main difficulty is that M_ℓ^o involves radiance measurements inside the layer. For $\ell = o$, we get from Eq. (9) the single-scattering albedo

$$1 - \omega_o = N_o/M_o^o \quad , \qquad (12)$$

where

$$N_o = \psi_1(\tau_1) - \psi_1(o) = \{F(\tau_1) - F(o)\}/2 \quad , \qquad (13)$$

and

$$M_o^o = \int_o^{\tau_1} \psi_o(\tau)\, d\tau = \frac{1}{2\pi} \int_o^{\tau_1} \Phi(\tau)\, d\tau \quad . \qquad (14)$$

Equation (12) is the integrated form of the well-known relation between the divergence of the net flux and the spherical flux

$$(1-\omega_o)\, \Phi(\tau) = \frac{dF}{d\tau} \quad . \qquad (15)$$

For $\ell = 1$ Eq. (9) yields

$$1 - \frac{\omega_o\, \beta_1}{3} = \frac{N_1}{3\, M_1^o} \quad , \qquad (16)$$

where

$$M_1^o = \int_o^{\tau_1} \psi_1(\tau)\, d\tau = \frac{1}{2\pi} \int_o^{\tau_1} F(\tau)\, d\tau \quad . \qquad (17)$$

Thus the important anisotropy factor $g = \beta_1/3$ of the phase function can be obtained only from measurements of the internal net flux and of the outgoing radiance. Another possibility is to express Δ_1 (that is β_1) in terms of Δ_o and of the spherical flux using M_o^o. For that purpose, multiply Eq. (6) by τ^α before integrating over τ ; we find

$$(2\ell+1)\, \Delta_\ell\, M_\ell^\alpha = (\ell+1) \int_o^{\tau_1} \tau^\alpha \frac{d}{d\tau} \psi_{\ell+1}(\tau)\, d\tau + \ell \int_o^{\tau_1} \tau^\alpha \frac{d}{d\tau} \psi_{\ell-1}(\tau)\, d\tau \; . \qquad (18)$$

With $\ell = o$ and $\alpha = 1$, we can integrate Eq. (18) to obtain

$$\Delta_o\, M_o^1 = \tau_1\, \psi_1(\tau_1) - \int_o^{\tau_1} \psi_1(\tau)\, d\tau \quad , \qquad (19)$$

and with Eq. (6) for $\ell = 1$, we can write

$$\Delta_o\, M_o^1 = \tau_1\, \psi_1(\tau_1) - \frac{N_1}{3\, \Delta_1} \quad , \qquad (20)$$

which defines Δ_1. In a similar way we can obtain Δ_2 (that is β_2) from Eq. (9) with $\ell = 2$ (α=o) ; thus

$$\Delta_2 = \frac{N_2}{5\, M_2^o} \quad , \qquad (21)$$

or from Eq. (18) with $\ell = 1$ and $\alpha = 1$

$$3\,\Delta_1\,M_1^1 = 2\,\tau_1\,\psi_2(\tau_1) + \tau_1\,\psi_o(\tau_1) - \frac{2\,N_2}{5\,\Delta_2} - \frac{N_o}{\Delta_o} \quad , \tag{22}$$

or from Eq. (18) with $\ell = o$ and $\alpha = 2$

$$\Delta_o\,M_o^2 = \tau_1^2\,\psi_1(\tau_1) - \frac{2}{3}\frac{\tau_1}{\Delta_1}\{2\,\psi_2(\tau_1) + \psi_o(\tau_1)\} + \frac{4}{15}\frac{N_2}{\Delta_1\,\Delta_2} + \frac{2}{3}\frac{N_o}{\Delta_o\,\Delta_1} \quad . \tag{23}$$

The extension to other Δ_ℓ is straightforward, and expressions in terms of the spherical flux or in terms of the net flux can be obtained. However the use of higher moments of the flux and of the previously computed values of $\Delta_\lambda(\lambda<\ell)$ must rapidly lead to an increase of numerical errors with the order ℓ.

SIMILARITY RELATIONS

We have so far assumed that the optical thickness τ_1 is known. Suppose now that it is not the case and choose an arbitrary values τ_1^*. Let us call

$$C = \tau_1\,/\tau_1^* \quad . \tag{24}$$

We have now built an "equivalent layer" (layer 2) where we can assume that C is at every depth the ratio of the actual extinction coefficient (or of the particles concentration) to its presently fixed value. The radiation field in layer 2 is assumed to be exactly the field observed in the real layer (layer 1). For this equivalent layer we will define a single scattering albedo $\bar{\omega}_o^*$ and phase function

$$p^*(\Theta) = \sum_{\ell=o}^{\infty} \beta_\ell^*\,P_\ell(\cos\Theta) \quad . \tag{25}$$

From Eq. (9), (10) and (24)

$$\Delta_\ell^* = 1 - \frac{\omega_o^*\,\beta_\ell^*}{2\ell+1} = C\Delta_\ell \quad , \tag{26}$$

that is

$$\bar{\omega}_o^* = 1 - (1-\bar{\omega}_o)\,C \quad , \tag{27}$$

$$\beta_\ell^* = \frac{C\bar{\omega}_o}{1-(1-\bar{\omega}_o)C}\,\beta_\ell + \frac{(1-C)}{1-(1-\bar{\omega}_o)C}(2\ell+1) \quad . \tag{28}$$

We will now demonstrate that this "equivalent layer" is identical to the real layer. The series of β_ℓ^* as defined by Eq. (28) is generaly divergent, unless C = 1. Using Eq. (28) in Eq. (25) the phase function of the equivalent layer can be written

$$p^*(\Theta) = (1-\frac{A}{2})\,p(\Theta) + A\delta\,(1-\cos\Theta) \quad , \tag{29}$$

where

$$A = \frac{2(1-C)}{1-(1-\bar{\omega}_o)C} \quad . \tag{30}$$

It is well known that the radiation scattered into a forward peak (represented by a Delta function) is physicaly equivalent to directly transmitted radiation ; therefore the problem can be handled by considering a layer 3 exactly equivalent to layer 2 with the following characteristics

$$\bar{\omega}'_o = \frac{\bar{\omega}^*_o(1 - \frac{A}{2})}{1 - \frac{\omega^*_o A}{2}} = \bar{\omega}_o \quad , \tag{31}$$

$$\tau'_1 = (1-\omega^*_o \frac{A}{2})\ \tau^*_1 = \tau_1 \quad , \tag{32}$$

$$p'(\Theta) = \sum_{\ell=o}^{\infty} \beta'_\ell \ \ P_\ell(\cos\Theta) \quad , \tag{33}$$

with

$$\beta'_\ell = \frac{(1-A/2)\beta_\ell}{1-A/2} = \beta_\ell \quad ; \tag{34}$$

where we have used Eq. (24), (27) (28) (29) and (30). Layer 3 is therefore identical to layer 1. Eq. (31) to (34) prove that all the characteristics (including the optical thickness) of the actual scattering layer can be retrieved from the complete knowledge of the diffuse radiation field. Of course this is true only in principle and its application is limited by the number of measurements and their experimental noise. For example it is well known that the radiation depends only very slightly on the high order β_ℓ when the field is very diffuse as it is the case for thick layers. Therefore on a practical point of view, many different "equivalent layers" corresponding to the same measured radiation with various τ_1 can be built. For the particular choice of $\beta^*_1 = o$ in Eq. (28) we can fix

$$C = 1\ /\ (1 - \frac{\beta_1\omega_o}{3}) \quad , \tag{35}$$

which gives

$$\tau^*_1 = \tau_1\ (1 - \frac{\beta_1\omega_o}{3}) \quad , \tag{36}$$

$$\omega^*_o = \frac{\omega_o(1-\beta_1/3)}{1-\omega_o\ \beta_1/3} \quad . \tag{37}$$

Eq. (36) and (37) are the Van de Hulst's similarity relations, which define an isotropic equivalent layer.

NUMERICAL TESTS

A first test of the principle of the method has been carried out using exact synthetic data. The radiance was computed by the spherical harmonics methods {5} with 6 digits accuracy. The various integrals over μ or τ were expressed analytically and computed with the same accuracy. Table 1 shows ω_o and the β_ℓ retrieved from Eq. (9) in comparison with the initial values. The second test simulates a sounding of the Venus clouds. The synthetic data were again obtained by the spherical harmonics method. The outgoing radiance was computed for n values of μ and m values of ϕ ; the integration over μ and ϕ was performed by using Gauss's quadrature scheme. Table 2 shows the effect of the number of directions and of the accuracy of the data on the inverted values. The net flux $F(\tau)$ was assumed to be known as a continuous function of τ, as in a real sounding, and the integrals M^α_1 have been computed by a trapezoidal rule with 50 points. The flux at great depth is multiplied by increasingly large values of τ^α; this is the first source of error and it limits the retrieval to β_1 and β_2 in the case of a thick, nearly conservative layer.

CONCLUSION

It has been shown that a scattering layer is completely defined by knowledge of the radiation field. The optical depth, the single-scattering albedo and the phase function could in principle be retrieved from a complete and infinitely accurate measurement of the radiation field. Practically, however, the retrieval is limited by the number of possible measurements and by their accuracy. Further numerical experiments are desired to fix the real possibilities of the method. In any case, it seems that the important anisotropy factor $g = \beta_1/3$ (related to the size of the particles) can be obtained accurately from a reasonable set of measurements, comprising the net flux inside the layer and the outgoing radiance.

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TABLE 1 - Comparison of Initial and Retrieval Parameters Using "Exact Synthetic" Data (τ_1 = 133.4)

	ω_0	β_1	β_2	β_3	β_4	β_{38}
Initial	0.997215	2.19756	3.27156	3.47972	4.23519	0.16046
Retrieval	0.997151	2.19544	3.26324	3.47020	4.22264	0.15957

TABLE 2 - Venus Cloud at λ = 0.36 µm and τ_1 = 82.2

	Exact	Retrieved Values			
		n = 8 m = 8	n = 4 m = 4	n = m = 8 5 % error	n = m = 8 10 % error
ω_0	0.969636	0.96881	0.96884	0.96883	0.9688
β_1	1.85396	1.86063	1.86185	1.89524	1.9275
β_2	2.44761	2.62524	2.61144	2.70712	2.7821
β_3	1.70766	3.31084	3.16965	/	/