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NOTE

AN EXACT EXPRESSION FOR THE WIEN DISPLACEMENT CONSTANT

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Abstract—A closed-form result for the Wien displacement constant is reported.

As discussed by e.g. Taylor and Glasstone[1], the Wien displacement constant A is given by

$$A = c_2/x, \tag{1}$$

where x is the non-zero real solution of

$$(5 - x)e^x = 5, \tag{2}$$

and the second radiation constant is $c_2 = ch/k$. Here we give an exact closed-form expression for x. If we let $y = x - 5$ we can write Eq. (2) as

$$ye^y = -5e^{-5} \tag{3}$$

and thus we deduce from Siewert and Burniston's general solution[2] of $z \exp z = a$ that

$$y = z + \left(\frac{z}{z+5}\right)[\ln 5 - 5 + \pi i - z - \log z]\exp\left\{-\frac{1}{\pi}\int_0^\infty [\theta(t) - \pi]\frac{dt}{t-z}\right\}, \tag{4}$$

where $\log z$ is that branch of the log function, in the plane cut along the positive real axis, such that $0 < \arg z < 2\pi$. In addition

$$\theta(t) = \tan^{-1}\left(\frac{\pi}{\ln 5 - 5 - t - \ln t}\right) \tag{5}$$

is continuous and such that $\theta(0) = 0$. Since Eq. (4) is valid for any z in the complex plane, we let $z = -5$ to obtain the simple result

$$x = 4\exp\left\{-\frac{1}{\pi}\int_0^\infty \left[\tan^{-1}\left(\frac{\pi}{\ln 5 - 5 - t - \ln t}\right) - \pi\right]\frac{dt}{t+5}\right\}. \tag{6}$$

We note that Garcia[3] has evaluated Eq. (6), by using an 80 point Gaussian quadrature scheme, to obtain $x = 4.96511\cdots$, which agrees with Birge's result,[4] and has solved Eq. (2) iteratively to obtain $x = 4.965114231744 28\cdots$.

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REFERENCES

1. H. S. Taylor and S. Glasstone, *A Treatise on Physical Chemistry*, Vol. 1. Van Nostrand, New York (1942).
2. C. E. Siewert and E. E. Burniston, *Math. Anal. Applic.* **43**, 626 (1973).
3. R. D. M. Garcia, private communication (1980).
4. R. T. Birge, *Rev. Mod. Phys.* **13**, 233 (1941).

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