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NOTE

A NOTE ON RADIATIVE TRANSFER

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Abstract—The formulation of radiative transfer problems with internal energy sources is discussed

1 DISCUSSION

In a recent publication Yuen and Wong[1] suggested that one of our early works[2] has only limited applicability in heat-transfer calculations because energy conservation was not considered and because a temperature distribution in the medium was assumed. Here we would like to point out that, with the exception of the limitation to isotropic scattering and from a mathematical point-of-view, the formulation we considered in that work[2] is sufficiently general so as to include the Yuen–Wong problem as a special case To be general, we start with a formulation that includes Lth order anisotropic scattering and that allows both specular and diffuse reflection at the surfaces Consider

$$\mu\frac{\partial}{\partial\tau}I(\tau,\mu)+I(\tau,\mu)=\frac{\omega}{2}\sum_{l=0}^{L}\beta_l P_l(\mu)\int_{-1}^{1}P_l(\mu')I(\tau,\mu')\,\mathrm{d}\mu'+S(\tau) \quad (1)$$

and the boundary conditions

$$I(L,\mu)=F_1(\mu)+\rho_1{}^sI(L,-\mu)+2\rho_1{}^d\int_{-1}^{1}I(L,-\mu')\mu'\,\mathrm{d}\mu',\qquad \mu>0, \quad (2a)$$

and

$$I(R,-\mu)=F_2(\mu)+\rho_2{}^sI(R,\mu)+2\rho_2{}^d\int_{-1}^{1}I(R,\mu')\mu'\,\mathrm{d}\mu',\qquad \mu>0, \quad (2b)$$

where $F_1(\mu)$ and $F_2(\mu)$ are given This problem, as recently reported,[3] can be solved for a given $S(\tau)$

We now investigate the type of problem considered by Yuen and Wong,[1] i e

$$\mu\frac{\partial}{\partial\tau}\psi(\tau,\mu)+\psi(\tau,\mu)=\frac{\omega}{2}\sum_{l=0}^{L}\gamma_l P_l(\mu)\int_{-1}^{1}P_l(\mu')\psi(\tau,\mu')\,\mathrm{d}\mu'+(1-\omega)I_b(\tau), \quad (3)$$

$$\psi(L,\mu)=F_1(\mu)+\rho_1{}^s\psi(L,-\mu)+2\rho_1{}^d\int_{0}^{1}\psi(L,-\mu')\mu'\,\mathrm{d}\mu',\qquad \mu>0, \quad (4a)$$

$$\psi(R,-\mu)=F_2(\mu)+\rho_2{}^s\psi(R,\mu)+2\rho_2{}^d\int_{0}^{1}\psi(R,\mu')\mu'\,\mathrm{d}\mu',\qquad \mu>0, \quad (4b)$$

and

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\int_{-1}^{1}\psi(\tau,\mu')\mu'\,\mathrm{d}\mu'=K(\tau), \quad (5)$$

where $F_1(\mu)$, $F_2(\mu)$ and the internal generation $K(\tau)$ are assumed given, but $I_b(\tau)$ is to be determined Equation (5) represents the requirement that the energy generated in the medium be dissipated only by the radiation process If we integrate Eq (3) over μ from -1 to 1 we find

$$\frac{d}{d\tau}\int_{-1}^{1} \psi(\tau, \mu)\mu \, d\mu + (1-\omega)\int_{-1}^{1} \psi(\tau, \mu)\, d\mu = 2(1-\omega)I_b(\tau) \tag{6}$$

or, after we use Eq (5),

$$2(1-\omega)I_b(\tau) = K(\tau) + (1-\omega)\int_{-1}^{1} \psi(\tau, \mu)\, d\mu \tag{7}$$

With Eq (7) we can eliminate the unknown $I_b(\tau)$ from Eq (3) to obtain

$$\mu\frac{\partial}{\partial\tau}\psi(\tau, \mu) + \psi(\tau, \mu) = \frac{1}{2}\int_{-1}^{1} \psi(\tau, \mu')\, d\mu' + \frac{1}{2}\sum_{l=1}^{L} \omega\gamma_l P_l(\mu)\int_{-1}^{1} P_l(\mu')\psi(\tau, \mu')\, d\mu' + \frac{1}{2}K(\tau), \tag{8}$$

which is to be solved subject to the boundary conditions given by Eqs (4)

It is therefore clear that in Eq (1) we can let $2S(\tau) = K(\tau)$, use $\beta_l = \omega\gamma_l$, $l \geq 1$, and set $\omega = 1$ to obtain a problem of the type considered by Yuen and Wong;[1] thus our formulation as given by Eqs (1) and (2) has enhanced applicability rather than limited utility in heat-transfer calculations

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