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ABSTRACT

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Radiative Transfer With Asymmetric Ground Reflection

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We use elementary considerations to carry out a Fourier decomposition of the radiation intensity for an atmosphere that has a ground defined by a general (asymmetric) reflection function, in order to represent better a remote-sensing problem.

We let $I(\tau,\mu,\varphi)$ denote the intensity (radiance) of the radiation field and utilize the equation of transfer[1] for a plane-parallel medium to model our atmosphere. We write

$$\mu\frac{\partial}{\partial\tau}I(\tau,\mu,\varphi)+I(\tau,\mu,\varphi)=\frac{\varpi}{4\pi}\int_{-1}^{1}\int_{0}^{2\pi}p(\cos\Theta)I(\tau,\mu',\varphi')d\varphi'd\mu' \tag{1}$$

where $\tau\in(0,\tau_0)$ is the optical variable and ϖ is the albedo for single scattering. In addition, $\mu\in[-1,1]$ and $\varphi\in[0,2\pi]$ are, respectively, the cosine of the polar angle (as measured from the *positive* τ axis) and the azimuthal angle which describe the direction of propagation of the radiation. We note also that the phase function $p(\cos\Theta)$ is represented by a finite Legendre polynomial expansion in terms of the scattering angle Θ.

We assume that the atmosphere is illuminated uniformly by a solar beam with a direction specified by (μ_0,φ_0), and so we seek a solution to Eq. (1) that satisfies the boundary conditions

$$I(0,\mu,\varphi)=\pi\delta(\mu-\mu_0)\delta(\varphi-\varphi_0) \tag{2}$$

and

$$I(\tau_0,-\mu,\varphi)=\int_0^{2\pi}\int_0^1 R(\mu',\mu,\varphi',\varphi)I(\tau_0,\mu',\varphi')\mu'd\mu'd\varphi' \tag{3}$$

for $\mu\in(0,1]$ and $\varphi\in[0,2\pi]$.

As we wish to include the possibility that there could be some phenomenon (*e.g.* rows of plants, or ocean waves) related to the ground that introduces a special direction into the problem, we make no assumptions regarding the symmetry of the function $R(\mu',\mu,\varphi',\varphi)$ that describes the reflection of radiation from some direction defined by the variables (μ',φ'), with $\mu'\in(0,1]$ and $\varphi'\in[0,2\pi]$, to another direction defined by the variables $(-\mu,\varphi)$, with $\mu\in(0,1]$ and $\varphi\in[0,2\pi]$. We then develop[2], for a general model of ground reflectance, a complete formulation for all of the component problems related to a Fourier decomposition of the radiation intensity, that is independent of the solution technique to be used.

In contrast to simpler problems, the intensity for this problem has both *sine* and *cosine* components that are coupled by way of the boundary conditions at the ground.

References

[1] S. Chandrasekhar, *Radiative Transfer*, Oxford University Press, London, 1950.
[2] L. B. Barichello, R. D. M. Garcia and C. E. Siewert, *JQSRT* **56**, 363 (1996).