A Computation of the X and Y Functions for a Nongrey Model with Complete Frequency Redistribution

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In this presentation we discuss a computation of the classic X and Y functions for use in the solution of a class of nongrey radiative-transfer problems defined for finite layers. We consider a model that allows for scattering with complete frequency redistribution (completely noncoherent scattering) and continuum absorption. In order to illustrate basic numerical aspects of our solution for the X and Y functions, we consider some test problems based on the Doppler and the Lorentz forms of the line-scattering coefficient.

Our problem is based on the equation of transfer written as

$$\mu\frac{\partial}{\partial\tau}I_x(\tau,\mu) = [\phi(x)+\beta][S_x(\tau) - I_x(\tau,\mu)]$$

where $S_x(\tau)$ is the source function,

$$[\phi(x)+\beta]S_x(\tau) = \frac{1}{2}\varpi\phi(x)\int_{-\infty}^{\infty}\phi(x')\int_{-1}^{1}I_{x'}(\tau,\mu')d\mu' dx' + [\rho\beta + (1-\varpi)\phi(x)]B(\tau),$$

and $B(\tau)$ is the Planck function. Here $\tau \in [0,\tau_0]$ is the optical variable, τ_0 is the optical thickness of the considered layer and $\mu \in [-1,1]$ is the *cosine* of the polar angle (as measured from the ***positive*** τ **axis**) that describes the direction of propagation of the radiation. In addition $\varpi \in [0,1]$ is the albedo for single scattering, $\beta \geq 0$ is the ratio of the continuum absorption coefficient to the average line coefficient, ρ is the ratio of the continuum source function to the Planck function and $\phi(x)$ is the line-scattering profile. We note that radiative-transfer problems in finite plane-parallel media are often expressed in terms of X and Y functions, and so our goal here is to compute those functions for this nongrey radiative-transfer model.

We consider the X and Y functions to be defined by the coupled singular-integral equations

$$\lambda(\eta)X(\eta) - \eta\,⨍_0^{\gamma}\Psi(\xi)X(\xi)\frac{d\xi}{\xi-\eta} + \eta e^{-\tau_0/\eta}\int_0^{\gamma}\Psi(\xi)Y(\xi)\frac{d\xi}{\xi+\eta} = 1$$

and

$$\lambda(\eta)Y(\eta) - \eta\,⨍_0^{\gamma}\Psi(\xi)Y(\xi)\frac{d\xi}{\xi-\eta} + \eta e^{-\tau_0/\eta}\int_0^{\gamma}\Psi(\xi)X(\xi)\frac{d\xi}{\xi+\eta} = e^{-\tau_0/\eta}$$

for $\eta \in [0,\gamma]$. We note that we use the symbol $⨍$ to denote that integrals are to be evaluated in the Cauchy principal-value sense. In addition $\gamma = 1/\beta$, $\Psi(\xi)$ is the characteristic function and

$$\lambda(\eta) = 1 + \eta\,⨍_{-\gamma}^{\gamma}\Psi(\xi)\frac{d\xi}{\xi-\eta}.$$

In this work we choose to use the F_N method to compute numerical solutions to Eqs. (3). We thus expand the X and Y functions in terms of a set of basis functions, substitute these approximations into Eqs. (3), consider the resulting equations at selected values of $\eta \in [0,\gamma]$ and solve a system of linear algebraic equations to find the required expansion coefficients in the approximate solutions. We note that the challenges of computing the desired X and Y functions for the considered radiative-transfer model are not insignificant!