THE SEARCHLIGHT PROBLEM FOR RADIATIVE TRANSFER

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A version of the discrete-ordinates method recently developed for radiative-transfer calculations is used along with numerical linear-algebra techniques and two-dimensional Fourier-transform procedures to establish the radiation flux and the z-component of the radiation current at all locations in a finite plane-parallel layer irradiated by a beam incident only at one point on one surface.

The problem is defined by the equation of transfer

$$\mu\frac{\partial}{\partial z}I(z,\boldsymbol{\rho},\boldsymbol{\Omega}) + \boldsymbol{\omega}\cdot\frac{\partial}{\partial\boldsymbol{\rho}}I(z,\boldsymbol{\rho},\boldsymbol{\Omega}) + I(z,\boldsymbol{\rho},\boldsymbol{\Omega}) = \frac{\varpi}{4\pi}\iint I(z,\boldsymbol{\rho},\boldsymbol{\Omega}')\,\mathrm{d}\boldsymbol{\Omega}'$$

for all z, $\boldsymbol{\rho}$ and $\boldsymbol{\Omega}$, and the boundary conditions

$$I[0,\boldsymbol{\rho},\boldsymbol{\Omega}(\mu,\phi)] = \frac{1}{2\pi\rho}\delta(\rho)\delta(\mu-\mu_0)\delta(\phi-\phi_0)$$

and

$$I[z_0,\boldsymbol{\rho},\boldsymbol{\Omega}(-\mu,\phi)] = 0$$

for $\mu \in (0,1]$ and $\phi \in [0,2\pi]$. We note that here $z \in [0,z_0]$ and $\boldsymbol{\rho}$, which lies in the $x-y$ plane, locate (in terms of mean-free paths) the position in the layer and that $\boldsymbol{\Omega}(\mu,\phi)$, with $\mu = \cos\theta$, is a unit vector that defines the direction of propagation. In addition, $\boldsymbol{\omega}$ is the projection of $\boldsymbol{\Omega}$ in the $x-y$ plane, $\boldsymbol{\Omega}(\mu_0,\phi_0)$ defines the direction of the incident beam and $\varpi < 1$ is the mean number of secondary particles per collision.

Considering that the foregoing equations define our basic problem, we seek to compute the radiation flux

$$I_0(z,\boldsymbol{\rho}) = \iint I(z,\boldsymbol{\rho},\boldsymbol{\Omega})\,\mathrm{d}\boldsymbol{\Omega}$$

and the z-component of the radiation current

$$I_1(z,\boldsymbol{\rho}) = \iint \mu I(z,\boldsymbol{\rho},\boldsymbol{\Omega})\,\mathrm{d}\boldsymbol{\Omega}$$

for $z \in [0,z_0]$ and all $\boldsymbol{\rho}$ of interest.

In addition to a general formulation basic to a beam that is incident at an oblique angle, for which the flux and current depend on three spatial variables, the Fourier transforms of the flux and current are inverted numerically for the two-dimensional case relevant to a normally incident beam. The reported numerical procedures, while computationally intensive, are thought to yield, for the considered test case, the radiation flux and the normal component of the radiation current with five figures of accuracy.