Viscous-slip, thermal-slip, and temperature-jump coefficients as defined by the linearized Boltzmann equation and the Cercignani–Lampis boundary condition

C. E. Siewert
Mathematics Department, North Carolina State University, Raleigh, North Carolina 27695-8205

(Received 18 November 2002; accepted 19 February 2003; published 6 May 2003)

A polynomial expansion procedure and an analytical discrete-ordinates method are used to evaluate the viscous-slip coefficient, the thermal-slip coefficient, and the temperature-jump coefficient as defined by a rigorous version of the linearized Boltzmann equation for rigid-sphere interactions and the Cercignani–Lampis boundary condition.
[DOI: 10.1063/1.1567284]

I. INTRODUCTION

While essentially all of the definitive numerical work in rarefied gas dynamics is based on the use of the classical Maxwell gas-surface interaction law (characterized by a single accommodation coefficient), there are some recent works[1–5] that make use of two accommodation coefficients α_t and α_n and the Cercignani–Lampis[6,7] gas-surface interaction law as an attempt to model better the effects of a bounding surface on the particle distribution function within the gas. In contrast to the Maxwell boundary condition which has the unique accommodation coefficient α for all physical properties, the Cercignani–Lampis (CL) condition[6,7] allows us to distinguish the accommodation of different properties. Physically, the quantity α_t is the accommodation coefficient of the tangential momentum, while the other quantity α_n describes the accommodation of the kinetic energy corresponding to the normal velocity. Since the CL boundary condition is based on the two mentioned accommodation coefficients, the use of this boundary condition yields the possibility of including better physics in the study of the basic problems of rarefied gas dynamics. In this work we report numerical results for the viscous-slip coefficient, the thermal-slip coefficient, and the temperature-jump coefficient as defined by the linearized Boltzmann equation for rigid-sphere interactions and the Cercignani–Lampis boundary condition.

In two recent works[8,9] a recently introduced polynomial expansion technique (relevant to the speed variable) and an analytical discrete-ordinates method[10] that has evolved from Chandrasekhar's work[11] in radiative transfer were used to solve the classical temperature-jump problem and a collection of basic flow problems all based on a rigorous form of the linearized Boltzmann equation for rigid-sphere interactions. While the two mentioned works defined (we believe) an improved standard of computational work in rarefied gas dynamics, the problems solved in those works were all based on the Maxwell gas-surface interaction law. And so here we report the additional work that is required to include the CL boundary condition in our analysis for the three most basic half-space problems in rarefied gas dynamics: Kramers' problem, the half-space problem of thermal creep, and the temperature-jump problem. We note that all the previous works[1–5] that include the CL boundary condition are based on low-level model equations, and so here, for the first time, we are able to include this boundary condition with a rigorous form of the linearized Boltzmann equation for rigid-sphere interactions.

II. GENERAL MATHEMATICAL FORMULATION

To start this work, we follow Pekeris[12] and consider the linearized Boltzmann equation written in terms of $h(\tau,\boldsymbol{c})$, a perturbation to the velocity distribution function, for rigid-sphere collisions as

$$S(\boldsymbol{c})+c\mu\frac{\partial}{\partial\tau}h(\tau,\boldsymbol{c})=\varepsilon L\{h\}(\tau,\boldsymbol{c}), \tag{1}$$

where

$$L\{h\}(\tau,\boldsymbol{c})=-\nu(c)h(\tau,\boldsymbol{c})+\int_0^\infty\int_{-1}^1\int_0^{2\pi}\boldsymbol{e}^{-c'^2}h(\tau,\boldsymbol{c}')\times K(\boldsymbol{c}':\boldsymbol{c})c'^2\,\mathrm{d}\chi'\,\mathrm{d}\mu'\,\mathrm{d}c'. \tag{2}$$

Here the scattering kernel is written in the expanded (Pekeris) form, viz.,

$$K(\boldsymbol{c}':\boldsymbol{c})=\frac{1}{4\pi}\sum_{n=0}^{\infty}\sum_{m=0}^{n}(2n+1)(2-\delta_{0,m})P_n^m(\mu')\times P_n^m(\mu)k_n(c',c)\cos m(\chi'-\chi), \tag{3}$$

where the $P_n^m(x)$ are the *normalized* Legendre functions. In addition,

$$\varepsilon=\sigma_0^2 n_0\pi^{1/2}l, \tag{4}$$

where l is (at this point) an unspecified mean-free path, n_0 is the density, and σ_0 is the scattering diameter of the gas particles. In this work, the spatial variable τ is measured in units of the mean-free path l and $c(2kT_0/m)^{1/2}$ is the magnitude of the particle velocity. Also, k is the Boltzmann constant, m is the mass of a gas particle and T_0 is a reference temperature. It should be noted that we have included in Eq. (1) an inhomogeneous driving term $S(\boldsymbol{c})$ that we will specify, along with an appropriate definition of the perturbation $h(\tau,\boldsymbol{c})$, for the half-space problem of thermal creep. We note that the functions $k_n(c',c)$ in Eq. (3) are the components in an expansion of the scattering law (for rigid-sphere collisions) reported by Pekeris[12] and

$$\nu(c)=\frac{2c^2+1}{c}\int_0^c \boldsymbol{e}^{-x^2}\,\mathrm{d}x+\boldsymbol{e}^{-c^2} \tag{5}$$

is the collision frequency. We use spherical coordinates $(c,\arccos\mu,\chi)$ to define the (dimensionless) velocity vector $\boldsymbol{c}$.

Following Pekeris,[12] Pekeris and Alterman[13] discussed the coefficients of viscosity and heat conduction and used the kernel functions $k_1(c',c)$ and $k_2(c',c)$ to define the Chapman–Enskog integral equations for viscosity and heat conduction. We write these equations here as

$$\nu(c)B(c)-\int_0^\infty \boldsymbol{e}^{-c'^2}B(c')k_2(c',c)c'^2\,\mathrm{d}c'=c^2 \tag{6}$$

and

$$\nu(c)A(c)-\int_0^\infty \boldsymbol{e}^{-c'^2}A(c')k_1(c',c)c'^2\,\mathrm{d}c'=c(c^2-5/2) \tag{7a}$$

with

$$\int_0^\infty \boldsymbol{e}^{-c^2}A(c)c^3\,\mathrm{d}c=0. \tag{7b}$$

Now, as noted previously,[13,14] if we wish to use a mean-free path based on the viscosity, i.e.,

$$l=l_p=(\mu_*/p_0)(2kT_0/m)^{1/2}, \tag{8}$$

where μ_* is the viscosity and $p_0=n_0kT_0$ is the pressure, then we should use in Eq. (1)

$$\varepsilon=\varepsilon_p=\frac{16}{15\pi^{1/2}}\int_0^\infty \boldsymbol{e}^{-c^2}B(c)c^4\,\mathrm{d}c, \tag{9}$$

where $B(c)$ is defined by Eq. (6). On the other hand, if we wish to use a mean-free path based on heat conduction,

$$l=l_t=[4\lambda_*/(5n_0k)][m/(2kT_0)]^{1/2}, \tag{10}$$

where λ_* is the heat-conduction coefficient, then in Eq. (1) we should use[13,15]

$$\varepsilon=\varepsilon_t=\frac{16}{15\pi^{1/2}}\int_0^\infty \boldsymbol{e}^{-c^2}A(c)c^5\,\mathrm{d}c, \tag{11}$$

where $A(c)$ is defined by Eq. (7). While the component kernel functions $k_n(c',c)$, for $n=1$ and 2 only, are required for the Chapman–Enskog equations for viscosity and heat conduction, we intend to use more of these component kernels in a truncated version of Eqs. (3). We note here that Pekeris and co-workers[16] have reported an ingenious set of expressions and recursion formulas that they used (along with a computer program) to obtain analytical results for the cases up to and including $k_8(c',c)$.

In this work we consider half-space problems, and so we supplement Eq. (1) with a boundary condition at the wall $(\tau=0)$. Since we are expressing the velocity vector in spherical coordinates, we note, in regard to Eq. (1), that

$$h(\tau,\boldsymbol{c})\Leftrightarrow h(\tau,c,\mu,\chi), \tag{12}$$

and so we express the boundary condition at the wall as

$$h(0,c,\mu,\chi)=\int_0^\infty\int_0^1\int_0^{2\pi} h(0,c',-\mu',\chi')\times R(\boldsymbol{c}':\boldsymbol{c})c'^2\,\mathrm{d}\chi'\,\mathrm{d}\mu'\,\mathrm{d}c' \tag{13}$$

for $\mu\in(0,1]$, $c\in[0,\infty)$ and all χ. For the case of the CL boundary condition we let $r(x)=(1-x^2)^{1/2}$, consider $\alpha_n\in[0,1]$ with $\alpha_t\in[0,2]$, and write

$$R(\boldsymbol{c}':\boldsymbol{c})=\frac{4c'\mu'}{a\alpha_n}S(c',\mu':c,\mu)T(\boldsymbol{c}':\boldsymbol{c})E_1(c',\mu':c,\mu), \tag{14}$$

where $a=\alpha_t(2-\alpha_t)$,

$$S(c',\mu':c,\mu)=\hat{I}_0[2(1-\alpha_n)^{1/2}cc'\mu\mu'/\alpha_n]\times E_2(c',\mu':c,\mu) \tag{15}$$

and

$$T(\boldsymbol{c}':\boldsymbol{c})=\frac{1}{2\pi}\exp\{-2cc'r(\mu)r(\mu')\times[|1-\alpha_t|-(1-\alpha_t)\cos(\chi'-\chi)]/a\}. \tag{16}$$

In addition

$$E_1(c',\mu':c,\mu)=\exp\{-[|1-\alpha_t|cr(\mu)-c'r(\mu')]^2/a\} \tag{17}$$

and

$$E_2(c',\mu':c,\mu)=\exp\{-[(1-\alpha_n)^{1/2}c\mu-c'\mu']^2/\alpha_n\}. \tag{18}$$

We find it convenient, from a computational point-of-view, to use in Eq. (15) and in general

$$\hat{I}_n(x)=I_n(x)\boldsymbol{e}^{-x} \tag{19}$$

in place of the modified Bessel functions $I_n(x)$. We note that we have arranged the components of the CL functions so as to avoid positive exponentials in our computation.

III. THE TEMPERATURE-JUMP PROBLEM

We have recently discussed[8] the solution of the temperature-jump problem as defined by the linearized Boltzmann equation for rigid-sphere interactions and the Maxwell boundary condition, and so our discussion here can be brief since we need mention only the (not exactly insignificant) complications introduced by the use of the Cercignani–Lampis boundary condition. While our basic problems are defined in terms of the velocity perturbation function $h(\tau,\boldsymbol{c})$, the quantities of interest generally are expressed in terms of integrals of this function. For example, here we seek the temperature perturbation

$$T(\tau)=\frac{2}{3\pi^{3/2}}\int_0^\infty\int_{-1}^1\int_0^{2\pi}e^{-c^2}(c^2-3/2)\times h(\tau,\boldsymbol{c})c^2\,\mathrm{d}\chi\,\mathrm{d}\mu\,\mathrm{d}c \tag{20}$$

which can be expressed as

$$T(\tau)=\frac{4}{3\pi^{1/2}}\int_0^\infty\int_{-1}^1 e^{-c^2}(c^2-3/2)\phi(\tau,c,\mu)c^2\,\mathrm{d}\mu\,\mathrm{d}c, \tag{21}$$

where

$$\phi(\tau,c,\mu)=\frac{1}{2\pi}\int_0^{2\pi}h(\tau,\boldsymbol{c})\,\mathrm{d}\chi \tag{22}$$

is an azimuthal average. For the temperature-jump problem we consider that $h(\tau,\boldsymbol{c})$ represents a perturbation from an absolute Maxwellian distribution, and so we express the velocity distribution function as

$$f(\tau,\boldsymbol{c})=f_0(c)[1+h(\tau,\boldsymbol{c})], \tag{23}$$

where

$$f_0(c)=n_0[m/(2\pi kT_0)]^{3/2}e^{-c^2}. \tag{24}$$

Our formulation of this problem does not have a driving term in Eq. (1), and so we can integrate Eqs. (l) and (13) over χ to find

$$c\mu\frac{\partial}{\partial\tau}\phi(\tau,c,\mu)=\varepsilon L_0\{\phi\}(\tau,c,\mu), \tag{25}$$

for $\tau>0$, $\mu\in[-1,1]$ and $c\in[0,\infty)$, and

$$\phi(0,c,\mu)=\int_0^\infty\int_0^1\phi(0,c',-\mu')R_0(c',\mu':c,\mu)\times c'^2\,\mathrm{d}\mu'\,\mathrm{d}c', \tag{26}$$

for $\mu\in(0,1]$ and $c\in[0,\infty)$. In regard to Eq. (25), we note that

$$L_0\{\phi\}(\tau,c,\mu)=-\nu(c)\phi(\tau,c,\mu)+\int_0^\infty\int_{-1}^1 e^{-c'^2}\phi(\tau,c',\mu')\times k_0(c',\mu':c,\mu)c'^2\,\mathrm{d}\mu'\,\mathrm{d}c', \tag{27}$$

where

$$k_0(c',\mu':c,\mu)=\int_0^{2\pi}K(\boldsymbol{c}':\boldsymbol{c})\,\mathrm{d}\chi \tag{28}$$

or

$$k_0(c',\mu':c,\mu)=\frac{1}{2}\sum_{n=0}^\infty(2n+1)P_n(\mu')\times P_n(\mu)k_n(c',c). \tag{29}$$

In addition,

$$R_0(c',\mu':c,\mu)=\int_0^{2\pi}R(\boldsymbol{c}':\boldsymbol{c})\,\mathrm{d}\chi. \tag{30}$$

Using Eq. (14), we deduce from Eq. (30) that

$$R_0(c',\mu':c,\mu)=\frac{4c'\mu'}{a\alpha_n}S(c',\mu':c,\mu)U_0(c',\mu':c,\mu), \tag{31}$$

where

$$U_0(c',\mu':c,\mu)=\hat{I}_0[2|1-\alpha_t|cc'r(\mu)r(\mu')/a]\times E_1(c',\mu':c,\mu). \tag{32}$$

Since Eqs. (25) and (26) are homogeneous, we must specify a driving term. We do this implicitly by requiring that $h(\tau,\boldsymbol{c})$ diverge as τ tends to infinity. More specifically, we impose the condition that the temperature perturbation satisfies the Welander condition,[17] viz.,

$$\lim_{\tau\to\infty}\frac{\mathrm{d}}{\mathrm{d}\tau}T(\tau)=\mathcal{K}, \tag{33}$$

where $\mathcal{K}$ is considered specified. The form of the temperature distribution here is the same as found before:[8]

$$T(\tau)=\mathcal{K}\left[\tau+\zeta+\sum_{j=3}^{J}A_jT_je^{-\varepsilon\tau/\nu_j}\right], \tag{34}$$

where the constants J, ν_j, and T_j are as defined previously.[8] Here the jump-coefficient ζ and the constants $\{A_j\}$ differ from those deduced before[8] only because we now are using the CL boundary condition instead of the Maxwell boundary condition. Later, we comment on the new work required to implement the CL boundary condition, and we report some selected results for the jump coefficient.

IV. THE FLOW PROBLEMS

First of all, in regard to Kramers' problem, we note that again Eq. (1) does not require an inhomogeneous term, and so the velocity distribution function is still given by Eq. (23). However, now the principal quantity of interest is the velocity profile:

$$u(\tau)=\frac{1}{\pi^{3/2}}\int_0^\infty\int_{-1}^1\int_0^{2\pi}e^{-c^2}h(\tau,\boldsymbol{c})\times c^3(1-\mu^2)^{1/2}\cos\chi\,\mathrm{d}\chi\,\mathrm{d}\mu\,\mathrm{d}c \tag{35}$$

or

$$u(\tau)=\frac{1}{\pi^{1/2}}\int_0^\infty\int_{-1}^1 e^{-c^2}\psi(\tau,c,\mu)c^3(1-\mu^2)\,\mathrm{d}\mu\,\mathrm{d}c, \tag{36}$$

where

$$\psi(\tau,c,\mu)=\frac{1}{\pi}(1-\mu^2)^{-1/2}\int_0^{2\pi}h(\tau,\boldsymbol{c})\cos\chi\,\mathrm{d}\chi. \tag{37}$$

At this point we can multiply Eqs. (1) and (13) by cos χ and integrate to find

$$c\mu\frac{\partial}{\partial\tau}\psi(\tau,c,\mu)=\varepsilon L_1\{\psi\}(\tau,c,\mu), \tag{38}$$

for $\tau>0$, $\mu\in[-1,1]$ and $c\in[0,\infty)$, and

$$r(\mu)\psi(0,c,\mu)=\int_0^\infty\int_0^1 r(\mu')\psi(0,c',-\mu')\times R_1(c',\mu':c,\mu)c'^2\,\mathrm{d}\mu'\,\mathrm{d}c', \tag{39}$$

for $\mu\in(0,1]$ and $c\in[0,\infty)$. In regard to Eq. (38), we note that

$$L_1\{\psi\}(\tau,c,\mu)=-\nu(c)\psi(\tau,c,\mu)+\int_0^\infty\int_{-1}^1 \boldsymbol{e}^{-c'^2}\psi(\tau,c',\mu')\times k_1(c',\mu':c,\mu)c'^2\,\mathrm{d}\mu'\,\mathrm{d}c', \tag{40}$$

where

$$k_1(c',\mu':c,\mu)=(1-\mu'^2)\sum_{n=1}^{\infty}\Pi_n(\mu')\Pi_n(\mu)k_n(c',c) \tag{41}$$

with

$$\Pi_n(\mu)=\left[\frac{2n+1}{2n(n+1)}\right]^{1/2}\frac{\mathrm{d}}{\mathrm{d}\mu}P_n(\mu),\quad n\geqslant 1. \tag{42}$$

In Eq. (39) the wall scattering function we require is defined by

$$R_1(c',\mu':c,\mu)\cos\chi'=\int_0^{2\pi}R(\boldsymbol{c}':\boldsymbol{c})\cos\chi\,\mathrm{d}\chi \tag{43}$$

from which we find

$$R_1(c',\mu':c,\mu)=\mathrm{signum}(1-\alpha_t)\frac{4c'\mu'}{a\alpha_n}\times S(c',\mu':c,\mu)U_1(c',\mu':c,\mu), \tag{44}$$

where

$$U_1(c',\mu':c,\mu)=\hat{I}_1[2|1-\alpha_t|cc'r(\mu)r(\mu')/a]\times E_1(c',\mu':c,\mu). \tag{45}$$

As with the temperature-jump problem, we see here that Eqs. (38) and (39) have no driving terms, so again we require that $h(\tau,\boldsymbol{c})$ diverge as τ tends to infinity, but at the same time the bulk velocity $u(\tau)$ should satisfy

$$\lim_{\tau\to\infty}\frac{\mathrm{d}}{\mathrm{d}\tau}u(\tau)=\mathcal{K}, \tag{46}$$

where the normalizing constant $\mathcal{K}$ is considered specified. As our solution of Kramers' problem, as based on Eq. (1) and the Maxwell boundary condition, was recently reported,[9] we can write the final form as

$$u_P(\tau)=\mathcal{K}\left[\tau+\zeta_P+\sum_{j=2}^{J}A_jN_j\boldsymbol{e}^{-\varepsilon\tau/\nu_j}\right], \tag{47}$$

where the constants J, ν_j, and N_j are as defined previously.[9] Now the viscous-slip coefficient ζ_P and the constants $\{A_j\}$ differ from those deduced before[9] only because we are using here the CL boundary condition instead of the Maxwell boundary condition.

For the case of thermal creep, the flow is caused by a constant temperature gradient in a direction parallel to the wall, and so it is helpful to linearize about a local Maxwellian rather than the absolute Maxwellian as was done in Eqs. (23) and (24). We follow Williams[18] and express the velocity distribution function as

$$f(\tau,\eta,\boldsymbol{c})=f_0(c)\{1+[(c^2-3/2)K_\eta+R_\eta]\eta+h(\tau,\boldsymbol{c})\}, \tag{48}$$

where $f_0(c)$ is given by Eq. (24) and we have expressed the imposed temperature and density variations as

$$T(\eta)=T_0(1+K_\eta\eta) \tag{49}$$

and

$$n(\eta)=n_0(1+R_\eta\eta). \tag{50}$$

We continue to use T_0 and n_0 as convenient reference values of the temperature and density, η is used to define (in terms of the mean-free path l) the direction of flow, and K_η and R_η are the constant gradients (in dimensionless units) of the temperature and density. For the problem of thermal creep we take $K_\eta=-R_\eta$, introduce $k_T=K_\eta$ and consider, since again we seek the bulk velocity profile, the balance equation

$$S_1(c,\mu)+c\mu\frac{\partial}{\partial\tau}\psi(\tau,c,\mu)=\varepsilon L_1\{\psi\}(\tau,c,\mu), \tag{51}$$

where the operator L_1 is defined by Eq. (40) and

$$S_1(c,\mu)=c(c^2-5/2)k_T. \tag{52}$$

So here we seek a bounded (as τ tends to infinity) solution of Eq. (51) that satisfies the boundary condition

$$r(\mu)\psi(0,c,\mu)=\int_0^\infty\int_0^1 r(\mu')\psi(0,c',-\mu')\times R_1(c',\mu':c,\mu)c'^2\,\mathrm{d}\mu'\,\mathrm{d}c', \tag{53}$$

for $\mu\in(0,1]$ and $c\in[0,\infty)$. Following our previous work[9] we can express the bulk velocity profile as

$$u_T(\tau)=k_T\left[\zeta_T+\sum_{j=2}^{J}A_jN_j\boldsymbol{e}^{-\varepsilon\tau/\nu_j}\right], \tag{54}$$

where ζ_T is the thermal-slip coefficient we report here for the case of the Cercignani–Lampis boundary condition.

Before proceeding to a discussion of our numerical results, we make note of several observations regarding the Cercignani–Lampis boundary condition as used here for the temperature-jump problem, the viscous-slip problem, and the thermal-slip problem as based on the linearized Boltzmann equation for a collection of rigid spheres. First of all we note that if we write $\alpha_t=1\pm x$, for $x\in[0,1]$, then $R_0(c',\mu':c,\mu)$, as given by Eq. (31), depends only on x and α_n. It follows that for the temperature-jump problem we need consider only $\alpha_t\in[0,1]$, with $\alpha_n\in[0,1]$. This simplification does not apply to the flow problems, and so for these cases we consider $\alpha_t\in[0,2]$, with $\alpha_n\in[0,1]$.

We list here some special cases we have deduced from the general functions used to define the CL boundary condition as used in this work:

$$\lim_{\alpha_n\to 0}\frac{1}{\alpha_n}S(c',\mu':c,\mu)=\frac{1}{2c\mu}\delta(c'\mu'-c\mu), \quad (55)$$

$$\lim_{\alpha_n\to 1}\frac{1}{\alpha_n}S(c',\mu':c,\mu)=\exp\{-c'^2\mu'^2\}, \quad (56)$$

$$\lim_{\alpha_t\to 0}\frac{1}{a}U_n(c',\mu':c,\mu)=\frac{1}{2cr(\mu)}\delta[c'r(\mu')-cr(\mu)], \quad (57)$$

$$\lim_{\alpha_t\to 1}\frac{1}{a}U_n(c',\mu':c,\mu)=\exp\{-c'^2(1-\mu'^2)\}\delta_{0,n}, \quad (58)$$

$$\lim_{\alpha_t\to 2}\frac{1}{a}U_n(c',\mu':c,\mu)=\frac{1}{2cr(\mu)}\delta[c'r(\mu')-cr(\mu)], \quad (59)$$

$$\lim_{\alpha_t\to 0}\lim_{\alpha_n\to 0}R_0(c',\mu':c,\mu)=\frac{1}{c^2}\delta(\mu'-\mu)\delta(c'-c), \quad (60)$$

$$\lim_{\alpha_t\to 2}\lim_{\alpha_n\to 0}R_0(c',\mu':c,\mu)=\frac{1}{c^2}\delta(\mu'-\mu)\delta(c'-c), \quad (61)$$

$$\lim_{\alpha_t\to 0}\lim_{\alpha_n\to 0}R_1(c',\mu':c,\mu)=\frac{1}{c^2}\delta(\mu'-\mu)\delta(c'-c) \quad (62)$$

and

$$\lim_{\alpha_t\to 2}\lim_{\alpha_n\to 0}R_1(c',\mu':c,\mu)=-\frac{1}{c^2}\delta(\mu'-\mu)\delta(c'-c). \quad (63)$$

As our basic analysis of the considered half-space problems is complete, we discuss briefly the modifications to our previous work[8,9] that are required to implement numerically the CL boundary condition.

V. NUMERICAL RESULTS

In regard to implementing our solution of the three problems considered in this work, we note that we have been able to use much of what we reported[8,9] for the case of the Maxwell boundary condition (a mixture of specular and diffuse reflection). That is to say, the use of our polynomial expansion technique (relative to the variable c) and our analytical discrete-ordinates method were used as before.[8,9] However to include the effects of the Cercignani–Lampis boundary condition required additional numerical work and led to a significantly more intensive computation. We see the complication introduced by the use of the CL boundary condition in Eqs. (26), (39), and (53) where repeated integrals must be evaluated numerically in order to define the linear system basic to determining the arbitrary constants in our general solution. While it is true that some of the special cases we considered involved delta "functions" that could be used to evaluate one of the integrals analytically, we found these cases also difficult and computer-time consuming since special arguments of some of the basic functions were required.

In this work we have five parameters that can be used to describe our solution: $L+1$ is the number of terms used in the Pekeris expansion of the scattering law, $K+1$ is the number of terms used in the polynomial expansion basic to the c variable, N is the number of Gauss points used in our analytical discrete-ordinates method, M is the number of Gauss points used to evaluate our input matrices, and I is the number of Gauss points used for evaluating integrals over the Cercignani–Lampis functions. The first four of these parameters are described in more detail in our previous work.[8,9] While some choices of the accommodation coefficients α_n and α_t led to calculations somewhat easier than others, and while some of the special cases proved more difficult than others, we have some confidence in our results obtained from the approximation space $\{8,200,30,30,200\}$ defined as $\{L,M,K,N,I\}$.

Before reporting our results for the viscous-slip coefficient, the thermal-slip coefficient, and the temperature-jump coefficient as defined by the CL boundary condition, we list

TABLE I. Viscous-slip ($\varepsilon=\varepsilon_p$), thermal-slip ($\varepsilon=\varepsilon_t$) and temperature-jump ($\varepsilon=\varepsilon_t$) coefficients for the Maxwell boundary condition.

α	ζ_P	ζ_T	ζ
0.1	1.704 78(1)	2.657 65(−1)	2.134 92(1)
0.2	8.172 48	2.744 50(−1)	1.025 15(1)
0.3	5.205 63	2.829 00(−1)	6.539 56
0.4	3.716 09	2.911 24(−1)	4.674 50
0.5	2.817 61	2.991 33(−1)	3.548 47
0.6	2.214 78	3.069 38(−1)	2.792 19
0.7	1.780 98	3.145 47(−1)	2.247 38
0.8	1.452 92	3.219 68(−1)	1.834 90
0.9	1.195 40	3.292 10(−1)	1.510 77
1	9.873 28(−1)	3.362 80(−1)	1.248 59

TABLE II. The viscous-slip ($\varepsilon=\varepsilon_p$) coefficient ζ_P for the Cercignani–Lampis boundary condition.

α_t	$\alpha_n=0$	$\alpha_n=0.25$	$\alpha_n=0.5$	$\alpha_n=0.75$	$\alpha_n=1$
0.25	6.3922	6.3645	6.3423	6.3232	6.3062
0.5	2.8161	2.7985	2.7841	2.7715	2.7602
0.75	1.6054	1.5970	1.5900	1.5838	1.5782
1	9.8733(−1)	9.8733(−1)	9.8733(−1)	9.8733(−1)	9.8733(−1)
1.25	6.0684(−1)	6.1452(−1)	6.2118(−1)	6.2721(−1)	6.3278(−1)
1.5	3.4532(−1)	3.6006(−1)	3.7305(−1)	3.8497(−1)	3.9609(−1)
1.75	1.5164(−1)	1.7290(−1)	1.9195(−1)	2.0964(−1)	2.2630(−1)
2	0	2.7335(−2)	5.2245(−2)	7.5624(−2)	9.7838(−2)

TABLE III. The thermal-slip ($\varepsilon=\varepsilon_t$) coefficient ζ_T for the Cercignani–Lampis boundary condition.

α_t	$\alpha_n=0$	$\alpha_n=0.25$	$\alpha_n=0.5$	$\alpha_n=0.75$	$\alpha_n=1$
0.25	2.6960(−1)	2.9049(−1)	3.1041(−1)	3.2950(−1)	3.4787(−1)
0.5	2.8905(−1)	3.0221(−1)	3.1503(−1)	3.2748(−1)	3.3958(−1)
0.75	3.1206(−1)	3.1834(−1)	3.2456(−1)	3.3068(−1)	3.3668(−1)
1	3.3628(−1)	3.3628(−1)	3.3628(−1)	3.3628(−1)	3.3628(−1)
1.25	3.5944(−1)	3.5369(−1)	3.4778(−1)	3.4183(−1)	3.3588(−1)
1.5	3.7915(−1)	3.6813(−1)	3.5663(−1)	3.4491(−1)	3.3306(−1)
1.75	3.9299(−1)	3.7723(−1)	3.6049(−1)	3.4323(−1)	3.2562(−1)
2	3.9894(−1)	3.7904(−1)	3.5751(−1)	3.3502(−1)	3.1183(−1)

TABLE IV. The temperature-jump ($\varepsilon=\varepsilon_t$) coefficient ζ for the Cercignani–Lampis boundary condition.

α_t	$\alpha_n=0$	$\alpha_n=0.25$	$\alpha_n=0.5$	$\alpha_n=0.75$	$\alpha_n=1$
0		1.6568(1)	7.6649	4.6889	3.2028
0.25	1.0151(1)	5.7318	3.7707	2.6655	1.9609
0.5	5.9030	3.8696	2.7282	2.0010	1.5015
0.75	4.7049	3.2245	2.3270	1.7284	1.3050
1	4.4041	3.0524	2.2161	1.6514	1.2486

in Table I our results[8,9] as defined by the linearized Boltzmann equation for rigid-sphere interactions and the Maxwell boundary condition. Note in Table I, and in subsequent tables, that while we use the mean-free path based on viscosity for the viscous-slip problem, we use the mean-free path based on thermal conductivity for the thermal-creep problem and the temperature-jump problem. The choice of a mean-free path is arbitrary here and so we made choices that were consistent with our previous work. In Tables II–IV we list our results for the viscous-slip coefficient, the thermal-slip coefficient, and the temperature-jump problem for selected values of the two CL accommodation coefficients α_t and α_n.

While we have no comparison results that are based on the linearized Boltzmann equation, we have used in our codes the relevant S-model approximations to the true scattering kernels to find essentially perfect agreement with S-model results for the slip coefficients[3] and an S-model code rewritten explicitly for the temperature-jump problem. Good agreement with Sharipov's S-model computations[4] for the slip and jump coefficients was also obtained. Having said that, we note that the special cases $\alpha_n=0$ and/or $\alpha_t=0$ were the most difficult for which to find stability, and so it is possible that our results for these special cases can be good only to four rather than five significant figures.

Finally we have concluded that to incorporate the Cercignani–Lampis boundary into a computation based on the linearized Boltzmann equation has required significant computational effort, but since the CL boundary condition offers the possibility of including better physics in the class of problems considered, the effort seems worthwhile.

ACKNOWLEDGMENTS

The author takes this opportunity to thank L. B. Barichello, N. J. McCormick, F. Sharipov, and A. M. Yacout for some helpful discussions regarding this (and other) work.

[1] F. Sharipov, "Application of the Cercignani–Lampis scattering kernel to calculations of rarefied gas flows. I. Plane flow between two parallel plates," Eur. J. Mech. B/Fluids **21**, 113 (2002).
[2] C. E. Siewert, "Generalized boundary conditions for the S-model kinetic equations basic to flow in a plane channel," J. Quant. Spectrosc. Radiat. Transf. **72**, 75 (2002).
[3] C. E. Siewert and F. Sharipov, "Model equations in rarefied gas dynamics: Viscous-slip and thermal-slip coefficients," Phys. Fluids **14**, 4123 (2002).
[4] F. Sharipov, "Application of the Cercignani–Lampis scattering kernel to calculations of rarefied gas flows. II. Slip and jump coefficients," to appear in Eur. J. Mech. B/Fluids.
[5] F. Sharipov, "Application of the Cercignani–Lampis scattering kernel to calculations of rarefied gas flows. III. Poiseuille flow and thermal-creep through a long tube," to appear in Eur. J. Mech. B/Fluids.
[6] C. Cercignani and M. Lampis, "Kinetic model for gas-surface interaction," Transp. Theory Stat. Phys. **1**, 101 (1971).
[7] I. Kuščer, J. Možina, and F. Križanič, "The Knudsen model of thermal accommodation," in *Rarefied Gas Dynamics 7*, edited by D. Oini (Editrice Tecnico Scientifica, Pisa, Italy, 1974), Vol. 1, p. 97.
[8] C. E. Siewert, "The linearized Boltzmann equation: A concise and accurate solution of the temperature-jump problem," J. Quant. Spectrosc. Radiat. Transf. **77**, 417 (2003).
[9] C. E. Siewert, "The linearized Boltzmann equation: Concise and accurate solutions to basic flow problems," ZAMP **54**, 273 (2003).
[10] L. B. Barichello and C. E. Siewert, "A discrete-ordinates solution for a non-grey model with complete frequency redistribution," J. Quant. Spectrosc. Radiat. Transf. **62**, 665 (1999).
[11] S. Chandrasekhar, *Radiative Transfer* (Oxford University Press, Oxford, 1950).
[12] C. L. Pekeris, "Solution of the Boltzmann–Hilbert integral equation," Proc. Natl. Acad. Sci. U.S.A. **41**, 661 (1955).
[13] C. L. Pekeris and Z. Alterman, "Solution of the Boltzmann–Hilbert integral equation. II. The coefficients of viscosity and heat conduction," Proc. Natl. Acad. Sci. U.S.A. **43**, 998 (1957).
[14] S. K. Loyalka and K. A. Hickey, "Plane Poiseuille flow: Near continuum results for a rigid sphere gas," Physica A **160**, 395 (1989).
[15] S. K. Loyalka and J. H. Ferziger, "Model dependence of the temperature slip coefficient," Phys. Fluids **11**, 1668 (1968).
[16] C. L. Pekeris, Z. Alterman, L. Finkelstein, and K. Frankowski, "Propagation of sound in a gas of rigid spheres," Phys. Fluids **5**, 1608 (1962).
[17] P. Welander, "On the temperature jump in a rarefied gas," Ark. Fys. **7**, 507 (1954).
[18] M. M. R. Williams, "A review of the rarefied gas dynamics theory associated with some classical problems in flow and heat transfer," ZAMP **52**, 500 (2001).