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A Legendre expansion and some exact solutions basic to the McCormack model for binary gas mixtures

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Abstract

A Legendre expansion of the scattering kernel, a conservation condition and some exact solutions are reported for the McCormack kinetic model that is used to describe a binary mixture of rarefied gases.

Keywords: Rarefied gas dynamics; McCormack model; Binary gas mixture

1. Introduction

During the past two years there has been considerable interest [1–9] in using the McCormack kinetic model [10] to describe binary gas mixtures in the general field of rarefied gas dynamics, and so in this brief work, we report a useful Legendre expansion of the scattering kernel, a conservation condition, and some exact solutions relevant to this model. These exact solutions are important components of the complete solutions to such classical problems as the temperature-jump problem, Kramers' problem (viscous-slip), and the thermal and diffusion slip problems.

We consider that the required functions $h_\alpha(x, \boldsymbol{v})$ for the two types of particles ($\alpha = 1$ and 2) denote perturbations from Maxwellian distributions for each species, i.e,

$$f_\alpha(x, \boldsymbol{v}) = f_{\alpha,0}(v)\big[1 + h_\alpha(x, \boldsymbol{v})\big], \tag{1}$$

where

$$f_{\alpha,0}(v) = n_\alpha(\lambda_\alpha/\pi)^{3/2}\,\mathrm{e}^{-\lambda_\alpha v^2}, \quad \lambda_\alpha = m_\alpha/(2kT_0). \tag{2}$$

Here k is the Boltzmann constant, m_α and n_α are the mass and the equilibrium density of the α-th species, x is the spatial variable (measured, for example, in cm), $\boldsymbol{v}$ is the particle velocity, and T_0 is a reference temperature. We follow Ref. [4] and note from McCormack's work [10] that the perturbations satisfy (for the case of variations in only one spatial variable) the coupled equations

$$c\mu\frac{\partial}{\partial x}h_\alpha(x, \boldsymbol{c}) + \omega_\alpha\gamma_\alpha h_\alpha(x, \boldsymbol{c}) = \omega_\alpha\gamma_\alpha\mathcal{L}_\alpha\{h_1, h_2\}(x, \boldsymbol{c}), \quad \alpha = 1, 2, \tag{3}$$

where we use a spherical coordinate system (c, μ, χ) to express the dimensionless velocity vector $\boldsymbol{c}$,

$$\omega_\alpha = \big[m_\alpha/(2kT_0)\big]^{1/2}, \tag{4}$$

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and the collision frequencies γ_α are to be defined. Here we write the integral operators as

$$\mathcal{L}_\alpha\{h_1, h_2\}(x, \boldsymbol{c}) = \frac{1}{\pi^{3/2}} \sum_{\beta=1}^{2} \int_0^\infty \int_{-1}^{1} \int_0^{2\pi} \mathrm{e}^{-c'^2} h_\beta(x, \boldsymbol{c}') K_{\beta,\alpha}(\boldsymbol{c}' : \boldsymbol{c}) c'^2 \, \mathrm{d}\chi' \, \mathrm{d}\mu' \, \mathrm{d}c', \tag{5}$$

where the kernels $K_{\beta,\alpha}(\boldsymbol{c}' : \boldsymbol{c})$ are taken from Ref. [4] and listed explicitly in Appendix A of this paper. We note that in obtaining Eq. (3) from the form given by McCormack [10], we have introduced the dimensionless velocity $\boldsymbol{c}$ differently in the two equations, i.e., for the case $\alpha = 1$ we used the transformation $\boldsymbol{c} = \omega_1 \boldsymbol{v}$, whereas for the case $\alpha = 2$ we used the transformation $\boldsymbol{c} = \omega_2 \boldsymbol{v}$. As we wish to work with a dimensionless spatial variable, we introduce

$$\tau = x/l_0, \tag{6}$$

where

$$l_0 = \frac{\mu v_0}{P_0} \tag{7}$$

is the mean-free path (based on viscosity) introduced by Sharipov and Kalempa [1]. Here, following Ref. [1], we write

$$v_0 = (2kT_0/m)^{1/2}, \tag{8}$$

where

$$m = \frac{n_1 m_1 + n_2 m_2}{n_1 + n_2}. \tag{9}$$

Continuing, we express the viscosity of the mixture in terms of the partial pressures P_α and the collision frequencies γ_α as [1]

$$\mu = P_1/\gamma_1 + P_2/\gamma_2, \tag{10}$$

where

$$\frac{P_\alpha}{P_0} = \frac{n_\alpha}{n_1 + n_2}, \tag{11}$$

$$\gamma_1 = \big[\Psi_1 \Psi_2 - \nu_{1,2}^{(4)} \nu_{2,1}^{(4)}\big]\big[\Psi_2 + \nu_{1,2}^{(4)}\big]^{-1}, \tag{12}$$

and

$$\gamma_2 = \big[\Psi_1 \Psi_2 - \nu_{1,2}^{(4)} \nu_{2,1}^{(4)}\big]\big[\Psi_1 + \nu_{2,1}^{(4)}\big]^{-1}. \tag{13}$$

Here definitions given in Appendix A have been used,

$$\Psi_1 = \nu_{1,1}^{(3)} + \nu_{1,2}^{(3)} - \nu_{1,1}^{(4)}, \tag{14}$$

and

$$\Psi_2 = \nu_{2,2}^{(3)} + \nu_{2,1}^{(3)} - \nu_{2,2}^{(4)}. \tag{15}$$

Finally, to compact our notation we introduce

$$\sigma_\alpha = \gamma_\alpha \omega_\alpha l_0 \tag{16}$$

or, more explicitly,

$$\sigma_\alpha = \gamma_\alpha \frac{n_1/\gamma_1 + n_2/\gamma_2}{n_1 + n_2} (m_\alpha/m)^{1/2}, \tag{17}$$

and so we rewrite Eq. (3) in terms of the τ variable as

$$c\mu \frac{\partial}{\partial \tau} h_\alpha(\tau, \boldsymbol{c}) + \sigma_\alpha h_\alpha(\tau, \boldsymbol{c}) = \sigma_\alpha \mathcal{L}_\alpha\{h_1, h_2\}(\tau, \boldsymbol{c}). \tag{18}$$

2. The Legendre expansion

From the explicit expressions listed in Appendix A, we see that we can write

$$K_{\alpha,\beta}(\boldsymbol{c}':\boldsymbol{c})=\sum_{n=0}^{2}\sum_{m=0}^{n}(2n+1)(2-\delta_{0,m})P_n^m(\mu')P_n^m(\mu)k_n^{(\alpha,\beta)}(c',c)\cos m(\chi'-\chi), \tag{19}$$

where

$$k_0^{(1,1)}(c',c)=1+(2/3)\big[1-2r^*\eta_{1,2}^{(1)}\big]\big(c'^2-3/2\big)\big(c^2-3/2\big), \tag{20a}$$

$$k_1^{(1,1)}(c',c)=(1/3)c'c\big\{2\big[1-\eta_{1,2}^{(1)}\big]-\eta_{1,2}^{(2)}\big(c'^2+c^2-5\big)+(4/5)\beta_1\big(c'^2-5/2\big)\big(c^2-5/2\big)\big\}, \tag{20b}$$

$$k_2^{(1,1)}(c',c)=(4/15)\varpi_1(c'c)^2, \tag{20c}$$

$$k_0^{(2,1)}(c',c)=(4/3)r^*\eta_{1,2}^{(1)}\big(c'^2-3/2\big)\big(c^2-3/2\big), \tag{21a}$$

$$k_1^{(2,1)}(c',c)=(1/3)c'c\big[r\big\{2\eta_{1,2}^{(1)}+\eta_{1,2}^{(2)}\big[r^2\big(c'^2-5/2\big)+c^2-5/2\big]\big\}+(4/5)\eta_{1,2}^{(6)}\big(c'^2-5/2\big)\big(c^2-5/2\big)\big], \tag{21b}$$

$$k_2^{(2,1)}(c',c)=(4/15)\eta_{1,2}^{(4)}(c'c)^2, \tag{21c}$$

$$k_0^{(1,2)}(c',c)=(4/3)s^*\eta_{2,1}^{(1)}\big(c'^2-3/2\big)\big(c^2-3/2\big), \tag{22a}$$

$$k_1^{(1,2)}(c',c)=(1/3)c'c\big[s\big\{2\eta_{2,1}^{(1)}+\eta_{2,1}^{(2)}\big[s^2\big(c'^2-5/2\big)+c^2-5/2\big]\big\}+(4/5)\eta_{2,1}^{(6)}\big(c'^2-5/2\big)\big(c^2-5/2\big)\big], \tag{22b}$$

$$k_2^{(1,2)}(c',c)=(4/15)\eta_{2,1}^{(4)}(c'c)^2, \tag{22c}$$

$$k_0^{(2,2)}(c',c)=1+(2/3)\big[1-2s^*\eta_{2,1}^{(1)}\big]\big(c'^2-3/2\big)\big(c^2-3/2\big), \tag{23a}$$

$$k_1^{(2,2)}(c',c)=(1/3)c'c\big\{2\big[1-\eta_{2,1}^{(1)}\big]-\eta_{2,1}^{(2)}\big(c'^2+c^2-5\big)+(4/5)\beta_2\big(c'^2-5/2\big)\big(c^2-5/2\big)\big\}, \tag{23b}$$

and

$$k_2^{(2,2)}(c',c)=(4/15)\varpi_2(c'c)^2. \tag{23c}$$

Here the *normalized* Legendre functions

$$P_n^m(\mu)=\left[\frac{(n-m)!}{(n+m)!}\right]^{1/2}\big(1-\mu^2\big)^{m/2}\frac{\mathrm{d}^m}{\mathrm{d}\mu^m}P_n(\mu),\quad n\geqslant m, \tag{24}$$

where $P_n(\mu)$ denotes one of Legendre polynomials, are such that

$$\int_{-1}^{1}P_n^m(\mu)P_{n'}^m(\mu)\,\mathrm{d}\mu=\left(\frac{2}{2n+1}\right)\delta_{n,n'}. \tag{25}$$

We now rewrite Eq. (18) as

$$c\mu\frac{\partial}{\partial\tau}\boldsymbol{H}(\tau,\boldsymbol{c})+\boldsymbol{\Sigma}\boldsymbol{H}(\tau,\boldsymbol{c})=\frac{1}{\pi^{3/2}}\boldsymbol{\Sigma}\int_0^\infty\int_{-1}^{1}\int_0^{2\pi}\mathrm{e}^{-c'^2}\boldsymbol{K}(\boldsymbol{c}':\boldsymbol{c})\boldsymbol{H}(\tau,\boldsymbol{c}')c'^2\,\mathrm{d}\chi'\,\mathrm{d}\mu'\,\mathrm{d}c' \tag{26}$$

where

$$\boldsymbol{H}(\tau,\boldsymbol{c})=\begin{bmatrix}h_1(\tau,\boldsymbol{c})\\ h_2(\tau,\boldsymbol{c})\end{bmatrix},\qquad \boldsymbol{K}(\boldsymbol{c}':\boldsymbol{c})=\begin{bmatrix}K_{1,1}(\boldsymbol{c}':\boldsymbol{c}) & K_{2,1}(\boldsymbol{c}':\boldsymbol{c})\\ K_{1,2}(\boldsymbol{c}':\boldsymbol{c}) & K_{2,2}(\boldsymbol{c}':\boldsymbol{c})\end{bmatrix}, \tag{27a,b}$$

and

$$\boldsymbol{\Sigma}=\mathrm{diag}\{\sigma_1\quad\sigma_2\}. \tag{27c}$$

When solving basic flow problems or problems based on temperature-density effects, it is sometimes possible to work with simplified balance equations obtained from certain moments (integrals) of Eq. (26). It is when this procedure can be used, that we find the expansion listed in Eq. (19) especially useful.

3. Solutions

In regard to solutions of Eq. (26), we have, first of all, the solutions that correspond to the conservation of number densities, momentum and kinetic energy (the collisional invariants). In our notation, these solutions take the forms

$$\boldsymbol{H}_1(\tau,\boldsymbol{c})=\begin{bmatrix}1\\0\end{bmatrix},\quad \boldsymbol{H}_2(\tau,\boldsymbol{c})=\begin{bmatrix}0\\1\end{bmatrix},\quad \boldsymbol{H}_3(\tau,\boldsymbol{c})=c^2\begin{bmatrix}1\\1\end{bmatrix},\quad \boldsymbol{H}_4(\tau,\boldsymbol{c})=c\mu\begin{bmatrix}1\\s\end{bmatrix}, \tag{28a–d}$$

$$\boldsymbol{H}_5(\tau,\boldsymbol{c})=c\big(1-\mu^2\big)^{1/2}\sin(\chi)\begin{bmatrix}1\\s\end{bmatrix},\quad \text{and}\quad \boldsymbol{H}_6(\tau,\boldsymbol{c})=c\big(1-\mu^2\big)^{1/2}\cos(\chi)\begin{bmatrix}1\\s\end{bmatrix}. \tag{28e,f}$$

In addition to the solutions given in Eqs. (28), we have found

$$\boldsymbol{H}_7(\tau,\boldsymbol{c})=\tau\boldsymbol{\Phi}_1(c)+c\mu\boldsymbol{F}_1(c)\quad \text{and}\quad \boldsymbol{H}_8(\tau,\boldsymbol{c})=\tau\boldsymbol{\Phi}_2(c)+c\mu\boldsymbol{F}_2(c), \tag{29a,b}$$

where $\boldsymbol{F}_1(c)$ and $\boldsymbol{F}_2(c)$ are expressed as

$$\boldsymbol{F}_\alpha(c)=\boldsymbol{F}_{\alpha,0}+\big(c^2-5/2\big)\boldsymbol{F}_{\alpha,2}. \tag{30}$$

Here $\boldsymbol{F}_{\alpha,0}$ and $\boldsymbol{F}_{\alpha,2}$ are solutions of the linear systems defined by

$$\boldsymbol{\mathcal{A}}\boldsymbol{F}_{\alpha,0}+\boldsymbol{\mathcal{B}}\boldsymbol{F}_{\alpha,2}=\boldsymbol{\Sigma}^{-1}\boldsymbol{\Phi}_{\alpha,0}, \tag{31a}$$

$$\boldsymbol{\mathcal{C}}\boldsymbol{F}_{\alpha,0}+\boldsymbol{\mathcal{D}}\boldsymbol{F}_{\alpha,2}=\boldsymbol{\Sigma}^{-1}\boldsymbol{\Phi}_{\alpha,2}, \tag{31b}$$

and

$$[0\quad 1]\boldsymbol{F}_{\alpha,0}=0. \tag{31c}$$

We list the coefficient matrices as

$$\boldsymbol{\mathcal{A}}=\begin{bmatrix}-\eta_{1,2}^{(1)} & r\eta_{1,2}^{(1)}\\ s\eta_{2,1}^{(1)} & -\eta_{2,1}^{(1)}\end{bmatrix},\qquad \boldsymbol{\mathcal{B}}=\frac{5}{4}\begin{bmatrix}-\eta_{1,2}^{(2)} & r^3\eta_{1,2}^{(2)}\\ s^3\eta_{2,1}^{(2)} & -\eta_{2,1}^{(2)}\end{bmatrix}, \tag{32a,b}$$

$$\boldsymbol{\mathcal{C}}=\frac{1}{2}\begin{bmatrix}-\eta_{1,2}^{(2)} & r\eta_{1,2}^{(2)}\\ s\eta_{2,1}^{(2)} & -\eta_{2,1}^{(2)}\end{bmatrix},\quad \text{and}\quad \boldsymbol{\mathcal{D}}=\begin{bmatrix}\beta_1-1 & \eta_{1,2}^{(6)}\\ \eta_{2,1}^{(6)} & \beta_2-1\end{bmatrix}. \tag{32c,d}$$

In addition,

$$\boldsymbol{\Phi}_\alpha(c)=\boldsymbol{\Phi}_{\alpha,0}+\big(c^2-5/2\big)\boldsymbol{\Phi}_{\alpha,2}, \tag{33}$$

with

$$\boldsymbol{\Phi}_{1,0}=\begin{bmatrix}c_1-1\\ c_1\end{bmatrix},\quad \boldsymbol{\Phi}_{1,2}=\begin{bmatrix}c_1\\ c_1\end{bmatrix},\quad \boldsymbol{\Phi}_{2,0}=\begin{bmatrix}c_2\\ c_2-1\end{bmatrix},\quad \text{and}\quad \boldsymbol{\Phi}_{2,2}=\begin{bmatrix}c_2\\ c_2\end{bmatrix}, \tag{34a,b,c,d}$$

where $c_\alpha=n_\alpha/(n_1+n_2)$. In regard to Eq. (26), we have also found two additional solutions that we write as

$$\boldsymbol{H}_9(\tau,\boldsymbol{c})=c\big(1-\mu^2\big)^{1/2}\sin(\chi)\boldsymbol{\mathcal{E}}(\tau,c,\mu) \tag{35a}$$

and

$$\boldsymbol{H}_{10}(\tau,\boldsymbol{c})=c\big(1-\mu^2\big)^{1/2}\cos(\chi)\boldsymbol{\mathcal{E}}(\tau,c,\mu), \tag{35b}$$

where

$$\boldsymbol{\mathcal{E}}=\begin{bmatrix}\sigma_1\tau-c\mu\\ s\sigma_1(\tau-c\mu/\sigma_2)\end{bmatrix}. \tag{36}$$

4. A flow condition

We can multiply Eq. (26) by $\exp\{-c^2\}c^2\,\mathrm{d}\chi\,\mathrm{d}\mu\,\mathrm{d}c$ and integrate to confirm that the (anticipated) flow condition

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\int_0^\infty\int_{-1}^1\int_0^{2\pi}\mathrm{e}^{-c^2}\boldsymbol{H}(\tau,\boldsymbol{c})c^3\mu\,\mathrm{d}\chi\,\mathrm{d}\mu\,\mathrm{d}c=\boldsymbol{0} \tag{37}$$

is valid.

5. Concluding remarks

In this brief work, we have reported some exact solutions to the McCormack kinetic model equations that have been used in numerous recent works to describe a binary mixture of dissimilar particles. Also included here is a (finite) spherical harmonics expansion of the scattering (matrix) kernel and a verification of an anticipated flow condition. While the solutions listed as Eqs. (28) follow directly from the conservation of number, energy, and momentum, the solutions defined by Eqs. (29) and (35) were much less easily foreseen. It is our opinion that the exact solutions reported in this work are especially important if we wish to obtain the correct asymptotic behavior when solving, say, the temperature-jump problem or Kramers' problem, both of which are defined for unbounded half-space media. To be clear, we note that the solutions listed as Eqs. (28), (29) and (35) do not define the complete solution for any specific physical problem, but moments (integrals) of these solutions can be used, as in Refs. [7–9], as exact components in an otherwise (discrete-ordinates, for example) approximate solution. In this way, the correct asymptotic behavior of the complete solution can be captured. Finally, this note is a prelude to a similar work now in progress where we seek to obtain similar results for a binary mixture of rigid spheres described by the linearized Boltzmann equation — a considerably more difficult task (especially in regard to the Legendre expansion of the scattering matrix).

Appendix A. Basic elements of the defining equations

Here we list some basic results that are required to define certain elements of the main text of this paper. First of all, in regard to Eq. (5), we note that

$$K_{\beta,\alpha}(\boldsymbol{c}':\boldsymbol{c}) = K^{(1)}_{\beta,\alpha}(\boldsymbol{c}':\boldsymbol{c}) + K^{(2)}_{\beta,\alpha}(\boldsymbol{c}':\boldsymbol{c}) + K^{(3)}_{\beta,\alpha}(\boldsymbol{c}':\boldsymbol{c}) + K^{(4)}_{\beta,\alpha}(\boldsymbol{c}':\boldsymbol{c}), \quad \alpha,\beta = 1,2, \tag{A.1}$$

where

$$K^{(1)}_{1,1}(\boldsymbol{c}':\boldsymbol{c}) = 1 + \big\{2\big[1-\eta^{(1)}_{1,2}\big] - \eta^{(2)}_{1,2}\big(c'^2 - 5/2\big)\big\}\boldsymbol{c}'\cdot\boldsymbol{c}, \tag{A.2}$$

$$K^{(2)}_{1,1}(\boldsymbol{c}':\boldsymbol{c}) = (2/3)\big[1 - 2r^*\eta^{(1)}_{1,2}\big]\big(c'^2-3/2\big)\big(c^2-3/2\big), \tag{A.3}$$

$$K^{(3)}_{1,1}(\boldsymbol{c}':\boldsymbol{c}) = 2\varpi_1\big[(\boldsymbol{c}'\cdot\boldsymbol{c})^2 - (1/3)c'^2c^2\big], \tag{A.4}$$

$$K^{(4)}_{1,1}(\boldsymbol{c}':\boldsymbol{c}) = \big[(4/5)\beta_1\big(c'^2-5/2\big) - \eta^{(2)}_{1,2}\big]\big(c^2-5/2\big)\boldsymbol{c}'\cdot\boldsymbol{c}, \tag{A.5}$$

$$K^{(1)}_{2,1}(\boldsymbol{c}':\boldsymbol{c}) = r\big\{2\eta^{(1)}_{1,2} + \eta^{(2)}_{1,2}\big[r^2\big(c'^2-5/2\big) + c^2 - 5/2\big]\big\}\boldsymbol{c}'\cdot\boldsymbol{c}, \tag{A.6}$$

$$K^{(2)}_{2,1}(\boldsymbol{c}':\boldsymbol{c}) = (4/3)r^*\eta^{(1)}_{1,2}\big(c'^2-3/2\big)\big(c^2-3/2\big), \tag{A.7}$$

$$K^{(3)}_{2,1}(\boldsymbol{c}':\boldsymbol{c}) = 2\eta^{(4)}_{1,2}\big[(\boldsymbol{c}'\cdot\boldsymbol{c})^2 - (1/3)c'^2c^2\big], \tag{A.8}$$

$$K^{(4)}_{2,1}(\boldsymbol{c}':\boldsymbol{c}) = (4/5)\eta^{(6)}_{1,2}\big(c'^2-5/2\big)\big(c^2-5/2\big)\boldsymbol{c}'\cdot\boldsymbol{c}, \tag{A.9}$$

$$K^{(1)}_{2,2}(\boldsymbol{c}':\boldsymbol{c}) = 1 + \big\{2\big[1-\eta^{(1)}_{2,1}\big] - \eta^{(2)}_{2,1}\big(c'^2 - 5/2\big)\big\}\boldsymbol{c}'\cdot\boldsymbol{c}, \tag{A.10}$$

$$K^{(2)}_{2,2}(\boldsymbol{c}':\boldsymbol{c}) = (2/3)\big[1 - 2s^*\eta^{(1)}_{2,1}\big]\big(c'^2-3/2\big)\big(c^2-3/2\big), \tag{A.11}$$

$$K^{(3)}_{2,2}(\boldsymbol{c}':\boldsymbol{c}) = 2\varpi_2\big[(\boldsymbol{c}'\cdot\boldsymbol{c})^2 - (1/3)c'^2c^2\big], \tag{A.12}$$

$$K^{(4)}_{2,2}(\boldsymbol{c}':\boldsymbol{c}) = \big[(4/5)\beta_2\big(c'^2-5/2\big) - \eta^{(2)}_{2,1}\big]\big(c^2-5/2\big)\boldsymbol{c}'\cdot\boldsymbol{c}, \tag{A.13}$$

$$K^{(1)}_{1,2}(\boldsymbol{c}':\boldsymbol{c}) = s\big\{2\eta^{(1)}_{2,1} + \eta^{(2)}_{2,1}\big[s^2\big(c'^2-5/2\big) + c^2 - 5/2\big]\big\}\boldsymbol{c}'\cdot\boldsymbol{c}, \tag{A.14}$$

$$K^{(2)}_{1,2}(\boldsymbol{c}':\boldsymbol{c}) = (4/3)s^*\eta^{(1)}_{2,1}\big(c'^2-3/2\big)\big(c^2-3/2\big), \tag{A.15}$$

$$K^{(3)}_{1,2}(\boldsymbol{c}':\boldsymbol{c}) = 2\eta^{(4)}_{2,1}\big[(\boldsymbol{c}'\cdot\boldsymbol{c})^2 - (1/3)c'^2c^2\big], \tag{A.16}$$

and

$$K^{(4)}_{1,2}(\boldsymbol{c}':\boldsymbol{c}) = (4/5)\eta^{(6)}_{2,1}\big(c'^2-5/2\big)\big(c^2-5/2\big)\boldsymbol{c}'\cdot\boldsymbol{c}. \tag{A.17}$$

Here we used

$$r=(m_1/m_2)^{1/2} \quad \text{and} \quad s=(m_2/m_1)^{1/2}, \tag{A.18}$$

along with

$$r^*=r^2/\big(1+r^2\big) \quad \text{and} \quad s^*=s^2/\big(1+s^2\big). \tag{A.19}$$

In addition,

$$\varpi_1=1+\eta_{1,1}^{(4)}-\eta_{1,1}^{(3)}-\eta_{1,2}^{(3)}, \tag{A.20}$$

$$\varpi_2=1+\eta_{2,2}^{(4)}-\eta_{2,2}^{(3)}-\eta_{2,1}^{(3)}, \tag{A.21}$$

$$\beta_1=1+\eta_{1,1}^{(6)}-\eta_{1,1}^{(5)}-\eta_{1,2}^{(5)}, \tag{A.22}$$

and

$$\beta_2=1+\eta_{2,2}^{(6)}-\eta_{2,2}^{(5)}-\eta_{2,1}^{(5)}, \tag{A.23}$$

where

$$\eta_{i,j}^{(k)}=\nu_{i,j}^{(k)}/\gamma_i. \tag{A.24}$$

Following McCormack [10], we write

$$\nu_{\alpha,\beta}^{(1)}=\frac{16}{3}\frac{m_{\alpha,\beta}}{m_\alpha}n_\beta\Omega_{\alpha,\beta}^{11}, \tag{A.25}$$

$$\nu_{\alpha,\beta}^{(2)}=\frac{64}{15}\left(\frac{m_{\alpha,\beta}}{m_\alpha}\right)^2 n_\beta\left(\Omega_{\alpha,\beta}^{12}-\frac{5}{2}\Omega_{\alpha,\beta}^{11}\right), \tag{A.26}$$

$$\nu_{\alpha,\beta}^{(3)}=\frac{16}{5}\left(\frac{m_{\alpha,\beta}}{m_\alpha}\right)^2 \frac{m_\alpha}{m_\beta}n_\beta\left(\frac{10}{3}\Omega_{\alpha,\beta}^{11}+\frac{m_\beta}{m_\alpha}\Omega_{\alpha,\beta}^{22}\right), \tag{A.27}$$

$$\nu_{\alpha,\beta}^{(4)}=\frac{16}{5}\left(\frac{m_{\alpha,\beta}}{m_\alpha}\right)^2 \frac{m_\alpha}{m_\beta}n_\beta\left(\frac{10}{3}\Omega_{\alpha,\beta}^{11}-\Omega_{\alpha,\beta}^{22}\right), \tag{A.28}$$

$$\nu_{\alpha,\beta}^{(5)}=\frac{64}{15}\left(\frac{m_{\alpha,\beta}}{m_\alpha}\right)^3 \frac{m_\alpha}{m_\beta}n_\beta\Gamma_{\alpha,\beta}^{(5)}, \tag{A.29}$$

and

$$\nu_{\alpha,\beta}^{(6)}=\frac{64}{15}\left(\frac{m_{\alpha,\beta}}{m_\alpha}\right)^3 \left(\frac{m_\alpha}{m_\beta}\right)^{3/2}n_\beta\Gamma_{\alpha,\beta}^{(6)}, \tag{A.30}$$

with

$$\Gamma_{\alpha,\beta}^{(5)}=\Omega_{\alpha,\beta}^{22}+\left(\frac{15m_\alpha}{4m_\beta}+\frac{25m_\beta}{8m_\alpha}\right)\Omega_{\alpha,\beta}^{11}-\left(\frac{m_\beta}{2m_\alpha}\right)\big(5\Omega_{\alpha,\beta}^{12}-\Omega_{\alpha,\beta}^{13}\big) \tag{A.31}$$

and, after a correction by Pan and Storvick [11],

$$\Gamma_{\alpha,\beta}^{(6)}=-\Omega_{\alpha,\beta}^{22}+\frac{55}{8}\Omega_{\alpha,\beta}^{11}-\frac{5}{2}\Omega_{\alpha,\beta}^{12}+\frac{1}{2}\Omega_{\alpha,\beta}^{13}. \tag{A.32}$$

In addition,

$$m_{\alpha,\beta}=m_\alpha m_\beta/(m_\alpha+m_\beta), \tag{A.33}$$

and the Ω functions are the Chapman–Cowling integrals [12,13] which for the case of rigid-sphere interactions take the simple forms

$$\Omega_{\alpha,\beta}^{12}=3\Omega_{\alpha,\beta}^{11}, \quad \Omega_{\alpha,\beta}^{13}=12\Omega_{\alpha,\beta}^{11} \quad \text{and} \quad \Omega_{\alpha,\beta}^{22}=2\Omega_{\alpha,\beta}^{11} \tag{A.34}$$

with

$$\Omega_{\alpha,\beta}^{11}=\frac{1}{4}\left(\frac{\pi kT_0}{2m_{\alpha,\beta}}\right)^{1/2}(d_\alpha+d_\beta)^2. \tag{A.35}$$

Here d_1 and d_2 are the diameters of the two types of particles, and, as noted in the main text of this work, k is the Boltzmann constant and T_0 is a reference temperature.

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