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Particular solutions of the linearized Boltzmann equation for a binary mixture of rigid spheres

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Abstract. Particular solutions that correspond to inhomogeneous driving terms in the linearized Boltzmann equation for the case of a binary mixture of rigid spheres are reported. For flow problems (in a plane channel) driven by pressure, temperature, and density gradients, inhomogeneous terms appear in the Boltzmann equation, and it is for these inhomogeneous terms that the particular solutions are developed. The required solutions for temperature and density driven problems are expressed in terms of previously reported generalized (vector-valued) Chapman–Enskog functions. However, for the pressure-driven problem (Poiseuille flow) the required particular solution is expressed in terms of two generalized Burnett functions defined by linear integral equations in which the driving terms are given in terms of the Chapman–Enskog functions. To complete this work, expansions in terms of Hermite cubic splines and a collocation scheme are used to establish numerical solutions for the generalized (vector-valued) Burnett functions.

Keywords. Rarefied gas dynamics, binary mixtures, rigid spheres, particular solutions, linearized Boltzmann equation, Chapman–Enskog and Burnett functions.

1. Introduction

The use of the Chapman–Enskog and Burnett functions for constructing particular solutions of the linearized Boltzmann equation for a single-species gas starts (as far as we can establish) with the work of Loyalka and Hickey [1]. The paper by Loyalka and Hickey [1] is focused on the problem of Poiseuille flow, but the authors also reported numerical results for (what they called) the Chapman–Enskog and Burnett functions. A discussion of these same functions (in a different notation) and a tabulation of numerical values is also available in a text by Sone [2]. Additional computations of these basic functions for the case of a single-species gas have been reported by Siewert [3] and Barichello, Rodrigues, and Siewert [4]. More relevant to this work is our recent computation [5] of the generalized Chapman–Enskog (vector-valued) functions relevant to the linearized Boltzmann equation for a binary mixture of rigid spheres.

2. Basic formulation

In this work we base our analysis of a binary gas mixture of rigid spheres on the linearized Boltzmann equation as formulated and utilized in Refs. [5–8]. While much of the formulation we use here was given in Ref. [5], we repeat some of that material since now we must account explicitly for the pressure gradient, the temperature gradient, and the density gradients that drive the flow. It is convenient to linearize our problem about local (rather that absolute) Maxwellian distributions, and so we start with the basic distribution functions written as

$$f_1(z,x,\boldsymbol{v}) = f_{1,0}(v) \quad \{1 + h_1(z, \lambda_1^{1/2}\boldsymbol{v}) + [(\lambda_1 v^2 - 5/2)K_T + K_P + (n_2/n)K_C]x\} \tag{2.1a}$$

and

$$f_2(z,x,\boldsymbol{v}) = f_{2,0}(v)\{1 + h_2(z, \lambda_2^{1/2}\boldsymbol{v}) + [(\lambda_2 v^2 - 5/2)K_T + K_P - (n_1/n)K_C]x\}, \tag{2.1b}$$

where

$$f_{\alpha,0}(v) = n_\alpha(\lambda_\alpha/\pi)^{3/2}\mathrm{e}^{-\lambda_\alpha v^2}, \quad \lambda_\alpha = m_\alpha/(2kT_0), \tag{2.2}$$

and $n = n_1 + n_2$. Here k is the Boltzmann constant, m_α and n_α are the mass and the equilibrium density of the α-th species, z is the spatial variable in the transverse, or cross-channel, direction, x is the spatial variable in the longitudinal direction, (both measured, for example, in *cm*), $\boldsymbol{v}$, with components v_x, v_y, v_z and magnitude v, is the particle velocity, and T_0 is a reference temperature. We note that the constants K_T, K_P, and K_C define respectively measures of the temperature, pressure, and density gradients that drive the flow (in the x direction).

In Refs. [5–7] the Boltzmann equation used was derived from linearizations about absolute Maxwellian distributions, rather than from linearizations as listed in Eqs. (2.1), and so now, as was done in Ref. [8], we must add inhomogeneous driving terms to the form of the balance equation used in Refs. [5–7]. Taking note of Eqs. (2.1), we start our work here with the linearized Boltzmann equation written as

$$\boldsymbol{S}(\boldsymbol{c}) + c\mu\frac{\partial}{\partial\tau}\boldsymbol{H}(\tau,\boldsymbol{c}) + \boldsymbol{V}(c)\boldsymbol{H}(\tau,\boldsymbol{c}) = \int \mathrm{e}^{-c'^2}\boldsymbol{\mathcal{K}}(\boldsymbol{c}' : \boldsymbol{c})\boldsymbol{H}(\tau,\boldsymbol{c}')\mathrm{d}^3c', \tag{2.3}$$

where

$$\boldsymbol{H}(\tau,\boldsymbol{c}) = \begin{bmatrix} h_1(\tau/\varepsilon_0,\boldsymbol{c}) \\ h_2(\tau/\varepsilon_0,\boldsymbol{c}) \end{bmatrix}, \tag{2.4}$$

and where we have introduced the dimensionless transverse spatial variable

$$\tau = z\varepsilon_0. \tag{2.5}$$

Here, in order to use the properties of both species of particles, we define our mean-free path in terms of

$$\varepsilon_0 = (n_1+n_2)\pi^{1/2}\Big(\frac{n_1 d_1 + n_2 d_2}{n_1 + n_2}\Big)^2, \tag{2.6}$$

where d_1 and d_2 are the diameters of the two species that define the mixture. Note that, in writing Eq. (2.3), we have followed Refs. [5–8] and have introduced a dimensionless velocity vector $\boldsymbol{c}$, with components c_x, c_y, c_z and magnitude c. However, rather than working with Cartesian coordinates, we use spherical coordinates $\{c, \theta, \phi\}$, with $\mu = \cos\theta$, to describe the dimensionless velocity vector, so that $c\mu$ is the component of the (dimensionless) velocity vector in the positive τ direction,

$$c_x = c(1-\mu^2)^{1/2}\cos\phi \tag{2.7}$$

is the component of the velocity in the direction of the flow, and

$$\boldsymbol{H}(\tau, \boldsymbol{c}) \Leftrightarrow \boldsymbol{H}(\tau, c, \mu, \phi).$$

We find we can write the inhomogeneous driving term in Eq. (2.3) as

$$\boldsymbol{S}(\boldsymbol{c}) = (c/\varepsilon_0)(1-\mu^2)^{1/2}\cos\phi \begin{bmatrix} (c^2-5/2)K_T + K_P + (n_2/n)K_C \\ (c^2-5/2)K_T + K_P - (n_1/n)K_C \end{bmatrix}. \tag{2.8}$$

In order to compact our presentation here, we do not list the definition of $\boldsymbol{V}(c)$ or of the matrix scattering kernel $\boldsymbol{\mathcal{K}}(\boldsymbol{c}' : \boldsymbol{c})$ that appear in Eq. (2.3); instead we consider that Ref. [8] is available where these quantities are defined explicitly (in the same notation used here).

Following Ref. [8], a work devoted to half-space flow (in the x direction) problems, we can, for the considered class of flow problems, write

$$\boldsymbol{H}(\tau, \boldsymbol{c}) = \boldsymbol{\Psi}(\tau, c, \mu)(1-\mu^2)^{1/2}\cos\phi, \tag{2.9}$$

and so we substitute Eq. (2.9) in Eq. (2.3) and use the Legendre expansion of the scattering kernel $\boldsymbol{\mathcal{K}}(\boldsymbol{c}' : \boldsymbol{c})$ that was reported in Ref. [5] to find

$$\begin{aligned} &\boldsymbol{\Upsilon}(c) + c\mu\frac{\partial}{\partial\tau}\boldsymbol{\Psi}(\tau, c, \mu) + \boldsymbol{V}(c)\boldsymbol{\Psi}(\tau, c, \mu) \\ &\qquad = \int_0^\infty \int_{-1}^1 \mathrm{e}^{-c'^2} f(\mu', \mu)\boldsymbol{\mathcal{K}}(c', \mu' : c, \mu)\boldsymbol{\Psi}(\tau, c', \mu')c'^2 \mathrm{d}\mu' \mathrm{d}c', \end{aligned} \tag{2.10}$$

where

$$f(\mu', \mu) = \left(\frac{1-\mu'^2}{1-\mu^2}\right)^{1/2}. \tag{2.11}$$

In addition,

$$\boldsymbol{\mathcal{K}}(c', \mu' : c, \mu)\cos\phi' = \int_0^{2\pi} \boldsymbol{\mathcal{K}}(\boldsymbol{c}' : \boldsymbol{c})\cos\phi \mathrm{d}\phi, \tag{2.12}$$

or

$$\boldsymbol{\mathcal{K}}(c', \mu' : c, \mu) = (1/2)\sum_{n=1}^\infty (2n+1)P_n^1(\mu')P_n^1(\mu)\boldsymbol{\mathcal{K}}_n(c', c), \tag{2.13}$$

where $P_n^1(x)$ is used to denote one of the normalized associated Legendre functions, and where the required definitions of the component kernels $\boldsymbol{\mathcal{K}}_n(c', c)$ are given explicitly in Refs. [5] and [8]. To complete Eq. (2.10), we note that the inhomogeneous driving term is

$$\boldsymbol{\Upsilon}(c) = (c/\varepsilon_0)\begin{bmatrix} (c^2 - 5/2)K_T + K_P + (n_2/n)K_C \\ (c^2 - 5/2)K_T + K_P - (n_1/n)K_C \end{bmatrix}. \tag{2.14}$$

3. The generalized Chapman–Enskog functions

For review, we list the defining equations, taken from Ref. [5], for (what we call) the generalized Chapman–Enskog (vector-valued) functions. In terms of the linear operators

$$\boldsymbol{\mathcal{L}}_n\{\boldsymbol{F}\}(c) = \boldsymbol{V}(c)\boldsymbol{F}(c) - \int_0^\infty \mathrm{e}^{-c'^2}\boldsymbol{\mathcal{K}}_n(c', c)\boldsymbol{F}(c')c'^2\mathrm{d}c', \tag{3.1}$$

we list the relevant integral equations as

$$\boldsymbol{\mathcal{L}}_1\{\boldsymbol{A}^{(1)}\}(c) = c\boldsymbol{\Phi}_1(c), \quad c \in [0, \infty), \tag{3.2a}$$

$$\boldsymbol{\mathcal{L}}_1\{\boldsymbol{A}^{(2)}\}(c) = c\boldsymbol{\Phi}_2(c), \quad c \in [0, \infty), \tag{3.2b}$$

and

$$\boldsymbol{\mathcal{L}}_2\{\boldsymbol{B}\}(c) = c^2\boldsymbol{\Phi}, \quad c \in [0, \infty), \tag{3.2c}$$

where

$$\boldsymbol{\Phi}_\alpha(c) = \boldsymbol{\Phi}_{\alpha,0} + (c^2 - 5/2)\boldsymbol{\Phi}_{\alpha,2}, \quad \alpha = 1, 2, \tag{3.3}$$

with

$$\boldsymbol{\Phi}_{1,0} = \begin{bmatrix} c_1 - 1 \\ c_1 \end{bmatrix}, \quad \boldsymbol{\Phi}_{1,2} = \begin{bmatrix} c_1 \\ c_1 \end{bmatrix}, \tag{3.4a,b}$$

$$\boldsymbol{\Phi}_{2,0} = \begin{bmatrix} c_2 \\ c_2 - 1 \end{bmatrix}, \quad \text{and} \quad \boldsymbol{\Phi}_{2,2} = \begin{bmatrix} c_2 \\ c_2 \end{bmatrix}, \tag{3.4c,d}$$

and where

$$\boldsymbol{\Phi} = \begin{bmatrix} 1 \\ a_{1,2} \end{bmatrix}. \tag{3.5}$$

In general, we use $c_\alpha = n_\alpha/(n_1 + n_2)$ and

$$a_{\alpha,\beta} = (m_\beta/m_\alpha)^{1/2}. \tag{3.6}$$

Because of the fact [5] that

$$\boldsymbol{A}_h(c) = \lambda c\begin{bmatrix} 1 \\ a_{1,2} \end{bmatrix}, \tag{3.7}$$

for any value of λ, is a solution of the homogeneous versions of Eqs. (3.2a) and (3.2b), we add $\boldsymbol{A}_h(c)$ to any solution we find of Eq. (3.2a) or (3.2b) and then

Table 1. The generalized Burnett functions for the Ne-Ar mixture for the case $c_1 = 0.1$

c	$D_1(c)$	$D_2(c)$	$E_1(c)$	$E_2(c)$
0.0	0.0	0.0	0.0	0.0
0.1	–3.536362(–1)	–2.390230(–1)	1.519502(–3)	8.230310(–4)
0.2	–6.889937(–1)	–4.682681(–1)	1.186713(–2)	6.512373(–3)
0.3	–9.899660(–1)	–6.784673(–1)	3.853333(–2)	2.158742(–2)
0.4	–1.244143	–8.613001(–1)	8.676809(–2)	4.992822(–2)
0.5	–1.443371	–1.009704	1.593159(–1)	9.457722(–2)
0.6	–1.583441	–1.118035	2.567032(–1)	1.576513(–1)
0.7	–1.663268	–1.182085	3.778279(–1)	2.403566(–1)
0.8	–1.683917	–1.198984	5.205987(–1)	3.430817(–1)
0.9	–1.647741	–1.167035	6.824673(–1)	4.655384(–1)
1.0	–1.557738	–1.085503	8.607996(–1)	6.069206(–1)
1.5	–4.089243(–1)	4.844150(–2)	1.921046	1.550966
2.0	1.673168	2.274307	3.125851	2.757838
2.5	4.469378	5.410527	4.376903	4.087493
3.0	7.838522	9.305649	5.633144	5.459620
3.5	1.168320(1)	1.384408(1)	6.878054	6.834072
4.0	1.593270(1)	1.893760(1)	8.105898	8.192771
4.5	2.053372(1)	2.451766(1)	9.315703	9.528949
5.0	2.544492(1)	3.052988(1)	1.050852(1)	1.084137(1)

determine the constants λ_1 and λ_2 so that our final solutions will satisfy the normalization conditions

$$\left[\begin{array}{cc} c_1 & c_2 \end{array}\right] \int_0^\infty \mathrm{e}^{-c^2} \boldsymbol{A}^{(\alpha)}(c) c^3 \mathrm{d}c = 0, \quad \alpha = 1, 2. \tag{3.8}$$

In Ref. [5] all three functions $\boldsymbol{A}^{(1)}(c), \boldsymbol{A}^{(2)}(c)$ and $\boldsymbol{B}(c)$ were expressed in terms of Hermite cubic splines and tabulated for some test cases. Henceforth, we consider these basic functions to be known.

4. Particular solutions for temperature and density gradients

In Ref. [8] we have already reported the particular solutions for the cases of temperature and density gradients. So we simply list these solutions here as

$$\boldsymbol{\Psi}_T(\tau, c, \mu) = -(1/\varepsilon_0)[\boldsymbol{A}^{(1)}(c) + \boldsymbol{A}^{(2)}(c)] K_T \tag{4.1}$$

and

$$\boldsymbol{\Psi}_C(\tau, c, \mu) = (1/\varepsilon_0)[c_2 \boldsymbol{A}^{(1)}(c) - c_1 \boldsymbol{A}^{(2)}(c)] K_C. \tag{4.2}$$

Table 2. The generalized Burnett functions for the Ne-Ar mixture for the case $c_1 = 0.5$

c	$D_1(c)$	$D_2(c)$	$E_1(c)$	$E_2(c)$
0.0	0.0	0.0	0.0	0.0
0.1	–2.659442(–1)	–1.699998(–1)	1.065199(–3)	5.862471(–4)
0.2	–5.192142(–1)	–3.330636(–1)	8.362924(–3)	4.649899(–3)
0.3	–7.483141(–1)	–4.825423(–1)	2.737917(–2)	1.547359(–2)
0.4	–9.438213(–1)	–6.123281(–1)	6.230267(–2)	3.597528(–2)
0.5	–1.098833	–7.170477(–1)	1.157797(–1)	6.858251(–2)
0.6	–1.208963	–7.921832(–1)	1.889669(–1)	1.151609(–1)
0.7	–1.272018	–8.341171(–1)	2.817887(–1)	1.769966(–1)
0.8	–1.287509	–8.401120(–1)	3.932826(–1)	2.548254(–1)
0.9	–1.256148	–8.082415(–1)	5.219370(–1)	3.488943(–1)
1.0	–1.179412	–7.372898(–1)	6.659686(–1)	4.590425(–1)
1.5	–1.855894(–1)	2.105991(–1)	1.555112	1.227384
2.0	1.648575	2.090584	2.603352	2.271090
2.5	4.139182	4.781919	3.712356	3.479596
3.0	7.162194	8.167974	4.837314	4.776960
3.5	1.063089(1)	1.215323(1)	5.958856	6.117681
4.0	1.448179(1)	1.666169(1)	7.069255	7.475935
4.5	1.866666(1)	2.163266(1)	8.166096	8.837543
5.0	2.314774(1)	2.701701(1)	9.249340	1.019500(1)

5. A particular solution for a pressure gradient

Considering pressure-driven flow, we follow Ref. [9] and seek a particular solution expressed as

$$\boldsymbol{\Psi}_P(\tau,c,\mu) = [1/(\varepsilon_0\varepsilon_p)]\{c\tau^2\boldsymbol{\Phi} - 2\mu\tau\boldsymbol{B}(c) + (1/5)\boldsymbol{D}(c) + [(5\mu^2-1)/5]\boldsymbol{E}(c)\}K_P, \quad (5.1)$$

where $\boldsymbol{B}(c)$ is defined by Eq. (3.2c) and where ε_p, $\boldsymbol{D}(c)$, and $\boldsymbol{E}(c)$ are to be determined. Substituting Eq. (5.1) into Eq. (2.10) for the considered case of pressure-driven flow, we find the integral equations

$$\boldsymbol{\mathcal{L}}_1\{\boldsymbol{D}\}(c) = 2c\boldsymbol{B}(c) - 5\varepsilon_p c\boldsymbol{\Gamma}, \quad c\in[0,\infty), \quad (5.2a)$$

and

$$\boldsymbol{\mathcal{L}}_3\{\boldsymbol{E}\}(c) = 2c\boldsymbol{B}(c), \quad c\in[0,\infty), \quad (5.2b)$$

where

$$\boldsymbol{\Gamma} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \quad (5.3)$$

Table 3. The generalized Burnett functions for the Ne-Ar mixture for the case $c_1 = 0.9$

c	$D_1(c)$	$D_2(c)$	$E_1(c)$	$E_2(c)$
0.0	0.0	0.0	0.0	0.0
0.1	–1.982432(–1)	–1.187171(–1)	7.091367(–4)	4.059196(–4)
0.2	–3.880557(–1)	–2.325293(–1)	5.599329(–3)	3.227514(–3)
0.3	–5.615392(–1)	–3.366787(–1)	1.849809(–2)	1.078330(–2)
0.4	–7.117687(–1)	–4.266888(–1)	4.259186(–2)	2.520670(–2)
0.5	–8.330812(–1)	–4.984751(–1)	8.024923(–2)	4.837497(–2)
0.6	–9.211891(–1)	–5.484256(–1)	1.329675(–1)	8.185992(–2)
0.7	–9.731412(–1)	–5.734477(–1)	2.014255(–1)	1.269022(–1)
0.8	–9.871745(–1)	–5.709840(–1)	2.856079(–1)	1.844077(–1)
0.9	–9.625069(–1)	–5.389996(–1)	3.849631(–1)	2.549627(–1)
1.0	–8.991174(–1)	–4.759493(–1)	4.985643(–1)	3.388633(–1)
1.5	–3.062103(–2)	3.315738(–1)	1.237215	9.535544(–1)
2.0	1.641545	1.957169	2.153169	1.847337
2.5	3.966367	4.335720	3.143863	2.941077
3.0	6.828188	7.382024	4.155751	4.167159
3.5	1.014218(1)	1.101716(1)	5.164157	5.477953
4.0	1.384499(1)	1.517402(1)	6.159023	6.842185
4.5	1.788802(1)	1.979668(1)	7.137287	8.239801
5.0	2.223305(1)	2.483861(1)	8.099071	9.657992

Again, because of the fact [5] that

$$\boldsymbol{D}_h(c) = \lambda c \begin{bmatrix} 1 \\ a_{1,2} \end{bmatrix}, \tag{5.4}$$

for any value of λ, is a solution of the homogeneous version of Eq. (5.2a), we intend to add $\boldsymbol{D}_h(c)$ to any solution we find of Eq. (5.2a), and then we determine the constant λ so that our final solution will satisfy the normalization condition

$$\begin{bmatrix} c_1 & c_2 \end{bmatrix} \int_0^\infty \mathrm{e}^{-c^2} \boldsymbol{D}(c) c^3 \mathrm{d}c = 0. \tag{5.5}$$

We proceed to determine the required constant ε_p. If we consider the inhomogeneous integral equation

$$\boldsymbol{\mathcal{L}}_n\{\boldsymbol{F}\}(c) = \boldsymbol{R}(c), \tag{5.6}$$

then we can use the fact [5] that

$$\boldsymbol{S}\boldsymbol{\mathcal{K}}^T(\boldsymbol{c} : \boldsymbol{c}') = \boldsymbol{\mathcal{K}}(\boldsymbol{c}' : \boldsymbol{c})\boldsymbol{S}, \tag{5.7}$$

Table 4. The generalized Burnett functions for the He-Xe mixture for the case $c_1 = 0.1$

c	$D_1(c)$	$D_2(c)$	$E_1(c)$	$E_2(c)$
0.0	0.0	0.0	0.0	0.0
0.1	–4.502358	–8.720557(–1)	2.326783(–2)	3.204674(–3)
0.2	–7.573805	–1.706968	1.366631(–1)	2.537995(–2)
0.3	–9.241140	–2.469359	3.216220(–1)	8.424910(–2)
0.4	–1.005329(1)	–3.127150	5.394149(–1)	1.952168(–1)
0.5	–1.040748(1)	–3.652705	7.710396(–1)	3.706070(–1)
0.6	–1.051824(1)	–4.023483	1.009547	6.192748(–1)
0.7	–1.049475(1)	–4.222207	1.252462	9.465817(–1)
0.8	–1.039257(1)	–4.236631	1.498825	1.354667
0.9	–1.024067(1)	–4.059018	1.748233	1.842918
1.0	–1.005484(1)	–3.685455	2.000519	2.408544
1.5	–8.838425	1.079521	3.304555	6.215502
2.0	–7.337471	1.023691(1)	4.681962	1.111142(1)
2.5	–5.624131	2.307254(1)	6.134873	1.650631(1)
3.0	–3.710048	3.897510(1)	7.662702	2.205899(1)
3.5	–1.594958	5.747466(1)	9.262514	2.760119(1)
4.0	7.229750(–1)	7.821198(1)	1.092956(1)	3.305986(1)
4.5	3.244387	1.009079(2)	1.265782(1)	3.841025(1)
5.0	5.968035	1.253411(2)	1.444045(1)	4.365066(1)

where the superscript T denotes the transpose operation and where

$$\boldsymbol{S} = \begin{bmatrix} c_2 & 0 \\ 0 & c_1 a_{1,2} \end{bmatrix}, \tag{5.8}$$

to deduce, in regard to Eq. (5.6), solvability condition(s) which we write as

$$\int_0^\infty \mathrm{e}^{-c^2} \boldsymbol{G}^T(c) \boldsymbol{S}^{-1} \boldsymbol{R}(c) c^2 \mathrm{d}c = 0, \tag{5.9}$$

where $\boldsymbol{G}(c)$ is any solution of the homogeneous equation

$$\boldsymbol{\mathcal{L}}_n\{\boldsymbol{F}\}(c) = \boldsymbol{0}. \tag{5.10}$$

We can now use the fact [5] that

$$\boldsymbol{G}(c) = c \begin{bmatrix} 1 \\ a_{1,2} \end{bmatrix} \tag{5.11}$$

is a solution of Eq. (5.10) for the case $n = 1$ to conclude from Eqs. (5.2a), (5.6), and (5.9) that

$$\varepsilon_p = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \boldsymbol{\varepsilon}_p, \tag{5.12}$$

Table 5. The generalized Burnett functions for the He-Xe mixture for the case $c_1 = 0.5$

c	$D_1(c)$	$D_2(c)$	$E_1(c)$	$E_2(c)$
0.0	0.0	0.0	0.0	0.0
0.1	–1.493783	–2.807519(–1)	5.083082(–3)	1.971211(–3)
0.2	–2.747772	–5.417248(–1)	3.608063(–2)	1.568308(–2)
0.3	–3.677910	–7.636366(–1)	1.032477(–1)	5.245047(–2)
0.4	–4.319351	–9.281652(–1)	2.036549(–1)	1.227696(–1)
0.5	–4.738506	–1.018345	3.302408(–1)	2.359865(–1)
0.6	–4.993860	–1.018881	4.765094(–1)	4.000478(–1)
0.7	–5.129006	–9.163554(–1)	6.375190(–1)	6.213437(–1)
0.8	–5.174873	–6.993449(–1)	8.096893(–1)	9.046452(–1)
0.9	–5.153170	–3.584350(–1)	9.904484(–1)	1.253126
1.0	–5.079209	1.138403(–1)	1.177943	1.668457
1.5	–4.220793	4.589183	2.180144	4.721769
2.0	–2.902413	1.263468(1)	3.247948	9.136197
2.5	–1.314326	2.399460(1)	4.362016	1.443864(1)
3.0	4.794627(–1)	3.828476(1)	5.518735	2.022459(1)
3.5	2.454194	5.513607(1)	6.717798	2.622278(1)
4.0	4.599536	7.423185(1)	7.958826	3.227241(1)
4.5	6.910775	9.530905(1)	9.240556	3.828642(1)
5.0	9.385251	1.181503(2)	1.056080(1)	4.422253(1)

where [5]

$$\boldsymbol{\varepsilon}_p = \frac{16}{15\pi^{1/2}} \int_0^\infty \mathrm{e}^{-c^2} \boldsymbol{B}(c) c^4 \mathrm{d}c. \tag{5.13}$$

To close this section, we note that each of the inhomogeneous driving terms used in Eqs. (3.2a) and (3.2b) satisfies the solvability condition listed as Eq. (5.9), for the case $n = 1$.

6. Hermite cubic splines and numerical results

In Ref. [5] we reported very explicitly all definitions required to establish our solution of Eqs. (3.2). As we use exactly the same expansions in terms of Hermite cubic splines and the same collocation procedure to solve Eqs. (5.2), we do not report again the details of these calculations.

At this point we can report some numerical results for two specific cases: the first is a mixture of Ne and Ar atoms, while the second is a mixture of He and Xe

Table 6. The generalized Burnett functions for the He-Xe mixture for the case $c_1 = 0.9$

c	$D_1(c)$	$D_2(c)$	$E_1(c)$	$E_2(c)$
0.0	0.0	0.0	0.0	0.0
0.1	−3.676171(−1)	3.019568(−1)	9.687980(−4)	1.434414(−3)
0.2	−7.134980(−1)	6.145593(−1)	7.617458(−3)	1.145953(−2)
0.3	−1.021428	9.483865(−1)	2.487811(−2)	3.858766(−2)
0.4	−1.282072	1.313886	5.623660(−2)	9.117648(−2)
0.5	−1.491248	1.721314	1.035945(−1)	1.773571(−1)
0.6	−1.647962	2.180673	1.675747(−1)	3.049682(−1)
0.7	−1.753146	2.701669	2.478885(−1)	4.814975(−1)
0.8	−1.808893	3.293659	3.436435(−1)	7.140320(−1)
0.9	−1.817950	3.965617	4.535876(−1)	1.009217
1.0	−1.783367	4.726102	5.762930(−1)	1.373226
1.5	−1.060625	1.012459(1)	1.332080	4.406470
2.0	3.562759(−1)	1.875799(1)	2.225551	9.797084
2.5	2.261964	3.118881(1)	3.176448	1.777522(1)
3.0	4.535714	4.773974(1)	4.150240	2.834577(1)
3.5	7.104270	6.854674(1)	5.134480	4.136247(1)
4.0	9.921148	9.360939(1)	6.126235	5.659115(1)
4.5	1.295571(1)	1.228330(2)	7.126307	7.375676(1)
5.0	1.618713(1)	1.560617(2)	8.136694	9.257548(1)

atoms. For the sake of our computations, we consider that the data

$$m_2 = 39.948 \quad m_1 = 20.183 \quad d_2/d_1 = 1.406 \quad \text{(Ne-Ar mixture)}$$

and

$$m_2 = 131.30 \quad m_1 = 4.0026 \quad d_2/d_1 = 2.226 \quad \text{(He-Xe mixture)}$$

are exact. It should be noted that the values of the masses used here are taken from Ref. [10], while the diameter ratios are those reported in Ref. [11]. We tabulate our results for these two cases in terms of c_1, the fractional concentration of the first particle. We note that the generalized Burnett (vector-valued) functions $\boldsymbol{D}(c)$ and $\boldsymbol{E}(c)$, as defined, depend only on three ratios: n_1/n_2, d_1/d_2 and m_1/m_2. We list in Tables 1–3 selected values of the two (vector-valued) functions for the Ne-Ar mixture, for three values of the concentration parameter: $c_1 = 0.1, 0.5, 0.9$. Similar results for the He-Xe mixture are given in Tables 4–6.

Finally, we note that we have compared results of our program with the single-gas results reported in the last two columns of Table 1 of Ref. [3], using three different ways of simulating the one-gas case with our formulation:

$$c_2 = 0$$

$$D_1(c) \to c^3 d(c) \quad E_1(c) \to c^3 e(c),$$

$$c_1 = 0$$

$$a_{2,1} D_2(c) \to c^3 d(c) \quad a_{2,1} E_2(c) \to c^3 e(c),$$

or

$$m_1/m_2 = 1 \quad d_1/d_2 = 1$$

$$D_1(c) \text{ and } D_2(c) \to c^3 d(c) \quad E_1(c) \text{ and } E_2(c) \to c^3 e(c).$$

We found perfect agreement with the results of Ref. [3], except the entry $c^3 d(c)$ with $c = 1.5$, for which we obtained $-7.351809(-4)$ instead of $-7.351806(-4)$.

7. Conclusions

While the particular solutions required for the problems of flow driven by temperature and concentration gradients and described by the linearized Boltzmann equation for a binary mixture of rigid-sphere gases were reported in Ref. [8], the particular solution for the case of Poiseuille flow in a plane channel has not been available. In providing our Eq. (5.1), we have now established this required particular solution. In addition to providing the mathematical result listed as Eq. (5.1), we have also reported (and illustrated with example computations) our numerical work with the generalized Burnett (vector-valued) functions that makes available numerical results relevant to Eq. (5.1). As a final comment, we note that Eq. (5.1) has been used recently [12] to solve well the problem of Poiseuille flow in a plane channel, as described by the linearized Boltzmann equation for a mixture of rigid-sphere gases.

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