European Journal of Mechanics B/Fluids 29 (2010) 181–191

Contents lists available at ScienceDirect

European Journal of Mechanics B/Fluids

journal homepage: www.elsevier.com/locate/ejmflu

Viscous-slip, thermal-slip, and temperature-jump coefficients based on the linearized Boltzmann equation (and five kinetic models) with the Cercignani–Lampis boundary condition

R.D.M. Garcia [a,*], C.E. Siewert [b]

[a] Instituto de Estudos Avançados, Rodovia dos Tamoios km 5.5, São José dos Campos, SP 12228-001, Brazil

[b] Mathematics Department, North Carolina State University, Raleigh, NC 27695-8205, USA

ARTICLE INFO

Article history:
Received 17 August 2009
Received in revised form
25 January 2010
Accepted 27 January 2010
Available online 7 February 2010

Keywords:
Rarefied gas dynamics
Linearized Boltzmann equation
Cercignani–Lampis boundary condition
Viscous-slip coefficient
Thermal-slip coefficient
Temperature-jump coefficient

ABSTRACT

A polynomial expansion procedure and the ADO (analytical discrete-ordinates) method are used to compute the viscous-slip coefficient, the thermal-slip coefficient, and the temperature-jump coefficient from the linearized Boltzmann equation (LBE) for rigid-sphere interactions and the Cercignani–Lampis (CL) boundary condition. These same quantities are also computed from five kinetic models, with the CL condition, and compared to the LBE result. Equivalent results for the LBE and the kinetic models, all based on the usual Maxwell boundary condition, are also reported.

1. Introduction

The importance of the use of temperature-jump and velocity-slip coefficients for improving the quality of calculations based on continuum equations has been pointed out by many authors (see, for example, Refs. [1–7]). Concerning the single-gas case which is treated in this work, we can cite several papers [8–17] that provide numerical results derived from solutions of the linearized Boltzmann equation (LBE) subject to Maxwell's (specular-diffuse) boundary condition for the mentioned coefficients, but we are aware of only one work [18] that uses the Cercignani–Lampis (CL) boundary condition [19] in this context. While other works [20–22] that make use of the CL boundary condition for the purpose of computing the temperature-jump and/or the velocity-slip coefficients are available, they are all based on model equations rather than the more rigorous LBE.

In Ref. [18], Siewert used a form of the LBE relevant to scattering where the gas particles are considered to be rigid spheres in order to compute the viscous-slip coefficient, the thermal-slip coefficient, and the temperature-jump coefficient. In that work [18], both the CL and Maxwell's boundary conditions were used, and a collection of numerical results obtained with the ADO (analytical discrete-ordinates) method [23] was reported. While several well studied numerical approximations were used in that previous paper [18], it was noted there that the use of only nine terms in a spherical-harmonics expansion of the rigid-sphere scattering kernel was a possible source of (thought to be) modest loss of accuracy. And so in this work we use a newly developed [24] numerical technique that can be used with confidence in high order for computing the expansion coefficients of the rigid-sphere kernel, along with the ADO method, to report improved numerical results for the three basic coefficients: viscous slip, thermal slip and temperature jump. And, as we have done in a recently completed work [25] on flow problems in a plane channel, we also include a comparison with numerical results for five kinetic models.

2. Basic formulation

We follow Ref. [18] and base our approach on the LBE for rigid-sphere scattering,

$$S(\boldsymbol{c}) + c\mu \frac{\partial}{\partial z} h(z, \boldsymbol{c}) + \varepsilon_0 \nu(c) h(z, \boldsymbol{c}) = \varepsilon_0 \int \mathrm{e}^{-c'^2} \mathcal{P}(\boldsymbol{c}' : \boldsymbol{c}) h(z, \boldsymbol{c}')\, \mathrm{d}^3 c', \tag{2.1}$$

where $h(z, \boldsymbol{c})$, the function to be determined, represents either a perturbation from an absolute Maxwellian distribution (in the

* Corresponding author. Tel.: +55 12 3922 6217; fax: +55 12 3944 1177.
E-mail addresses: rdgarcia@ieav.cta.br, rdmgarcia@uol.com.br (R.D.M. Garcia).

0997-7546/$ – see front matter
doi:10.1016/j.euromechflu.2010.01.005

cases of the temperature-jump and viscous-slip problems) or a perturbation from a local Maxwellian distribution (in the case of the thermal-creep problem). In general, the perturbation may depend on z, a spatial variable that measures the distance from the boundary into the gas, and the dimensionless velocity variable

$$\boldsymbol{c} = \lambda^{1/2}\boldsymbol{v}, \quad \lambda = m/(2kT_0), \tag{2.2}$$

where $\boldsymbol{v}$ is the velocity, m is the mass of a particle, k is the Boltzmann constant, and T_0 is a reference temperature. Continuing to define the quantities that appear in Eq. (2.1), we have the collision frequency

$$\nu(c) = \frac{2c^2+1}{c}\int_0^c \mathrm{e}^{-x^2}\,\mathrm{d}x + \mathrm{e}^{-c^2}, \tag{2.3}$$

where c is the magnitude of $\boldsymbol{c}$, the rigid-sphere scattering kernel [26]

$$\mathcal{P}(\boldsymbol{c}' : \boldsymbol{c}) = \frac{1}{\pi}\left(\frac{2}{|\boldsymbol{c}'-\boldsymbol{c}|}\exp\left\{\frac{|\boldsymbol{c}'\times\boldsymbol{c}|^2}{|\boldsymbol{c}'-\boldsymbol{c}|^2}\right\} - |\boldsymbol{c}'-\boldsymbol{c}|\right), \tag{2.4}$$

and

$$\varepsilon_0 = n\pi^{1/2}d^2, \tag{2.5}$$

where n is the particle density at equilibrium and d is the diameter of the particles. We note that the inhomogeneous term $S(\boldsymbol{c})$ in Eq. (2.1) is relevant only to the thermal-creep problem [18] and that spherical coordinates $\{c, \theta, \phi\}$, with $\mu = \cos\theta$, are used to describe the dimensionless velocity vector $\boldsymbol{c}$, so that

$$h(z, \boldsymbol{c}) \Leftrightarrow h(z, c, \mu, \phi).$$

As in Ref. [18], we consider that the interaction of the gas with the bounding surface (located at $z = 0$) is described by the CL boundary condition

$$h(0, c, \mu, \phi) = \int_0^\infty\int_0^1\int_0^{2\pi} h(0, c', -\mu', \phi') \times R(c', -\mu', \phi' : c, \mu, \phi)c'^2\,\mathrm{d}\phi'\,\mathrm{d}\mu'\,\mathrm{d}c', \tag{2.6}$$

for $c \in [0, \infty)$, $\mu \in (0, 1]$, and $\phi \in [0, 2\pi]$, where the CL kernel can be written as [18,25]

$$R(c', -\mu', \phi' : c, \mu, \phi) = \frac{2c'\mu'}{\widehat{\alpha}\alpha_n\pi}S(c', -\mu' : c, \mu) \times T(c', -\mu', \phi' : c, \mu, \phi). \tag{2.7}$$

Here, $\widehat{\alpha} = \alpha_t(2-\alpha_t)$,

$$S(c', -\mu' : c, \mu) = \exp\{-[(c'\mu')^2 + (1-\alpha_n)(c\mu)^2]/\alpha_n\} \times I_0[2(1-\alpha_n)^{1/2}c'\mu'c\mu/\alpha_n], \tag{2.8a}$$

and

$$T(c', -\mu', \phi' : c, \mu, \phi) = E(c', \mu' : c, \mu) \times \exp\{-2c'r(\mu')cr(\mu)[|1-\alpha_t| - (1-\alpha_t)\cos(\phi'-\phi)]/\widehat{\alpha}\}, \tag{2.8b}$$

with

$$E(c', \mu' : c, \mu) = \exp\{-[|1-\alpha_t|cr(\mu) - c'r(\mu')]^2/\widehat{\alpha}\}. \tag{2.9}$$

In these expressions, $\alpha_t \in [0, 2]$ represents the accommodation coefficient of tangential momentum and $\alpha_n \in [0, 1]$ that of the kinetic energy due to the normal component of the velocity. In addition, $I_n(x)$ is used to denote the nth-order modified Bessel function of the first kind and $r(x) = (1-x^2)^{1/2}$.

3. The temperature-jump problem

As mentioned in Section 2, $h(z, \boldsymbol{c})$ is taken to represent a perturbation from an absolute Maxwellian distribution in the case of the temperature-jump problem, and so the velocity distribution function is

$$f(z, \boldsymbol{v}) = f_0(v)[1 + h(z, \boldsymbol{c})], \tag{3.1}$$

where v is the magnitude of $\boldsymbol{v}$ and

$$f_0(v) = n(\lambda/\pi)^{3/2}\mathrm{e}^{-\lambda v^2}. \tag{3.2}$$

Introducing the dimensionless spatial variable

$$\tau = z\varepsilon_0, \tag{3.3}$$

where ε_0 is defined by Eq. (2.5), and noting that the perturbation $h(z, c, \mu, \phi)$ does not depend on ϕ for this problem, we can use

$$h(\tau/\varepsilon_0, \boldsymbol{c}) = \psi_0(\tau, c, \mu) \tag{3.4}$$

in Eq. (2.1) for $S(\boldsymbol{c}) = 0$ and in Eq. (2.6) and integrate both of these equations over ϕ, to show that the temperature-jump problem with the CL boundary condition can be reduced to solving

$$c\mu\frac{\partial}{\partial\tau}\psi_0(\tau, c, \mu) + \nu(c)\psi_0(\tau, c, \mu) = \int_0^\infty\int_{-1}^1 \mathrm{e}^{-c'^2}\mathcal{P}_0(c', \mu' : c, \mu)\psi_0(\tau, c', \mu')c'^2\,\mathrm{d}\mu'\,\mathrm{d}c' \tag{3.5}$$

for $\tau \in (0, \infty)$, $c \in [0, \infty)$, and $\mu \in [-1, 1]$, subject to

$$\psi_0(0, c, \mu) = \int_0^\infty\int_0^1 \psi_0(0, c', -\mu')R_0(c', -\mu' : c, \mu)c'^2\,\mathrm{d}\mu'\,\mathrm{d}c' \tag{3.6}$$

for $c \in [0, \infty)$ and $\mu \in (0, 1]$. We note that in Eq. (3.5) the kernel $\mathcal{P}_0(c', \mu' : c, \mu)$ can be written as

$$\mathcal{P}_0(c', \mu' : c, \mu) = (1/2)\sum_{n=0}^\infty (2n+1)P_n(\mu')P_n(\mu)\mathcal{P}^{(n)}(c', c), \tag{3.7}$$

where $P_n(x)$ denotes a Legendre polynomial and $\{\mathcal{P}^{(n)}(c', c)\}$ are expansion coefficients [24]. Now, with regard to Eq. (3.6) we have [18]

$$R_0(c', -\mu' : c, \mu) = \frac{4c'\mu'}{\widehat{\alpha}\alpha_n}S(c', -\mu' : c, \mu)U_0(c', -\mu' : c, \mu), \tag{3.8}$$

where $S(c', -\mu' : c, \mu)$ is given by Eq. (2.8a), and, in general,

$$U_n(c', -\mu' : c, \mu) = \exp\{-[c'^2r^2(\mu') + (1-\widehat{\alpha})c^2r^2(\mu)]/\widehat{\alpha}\} \times I_n[2(1-\widehat{\alpha})^{1/2}c'r(\mu')cr(\mu)/\widehat{\alpha}]. \tag{3.9}$$

Equation (3.8) is valid for all of the allowed values of $\widehat{\alpha}$ and α_n, except the following special cases [18]:

$$\lim_{\widehat{\alpha}\to 0} R_0(c', -\mu' : c, \mu) = \frac{2\mu'}{\alpha_n r(\mu')}S(c', -\mu' : c, \mu) \times \delta[c'r(\mu') - cr(\mu)], \tag{3.10a}$$

$$\lim_{\alpha_n\to 0} R_0(c', -\mu' : c, \mu) = \frac{2}{\widehat{\alpha}}U_0(c', -\mu' : c, \mu)\delta(c'\mu' - c\mu), \tag{3.10b}$$

and

$$\lim_{\widehat{\alpha}\to 0}\lim_{\alpha_n\to 0} R_0(c', -\mu' : c, \mu) = \frac{1}{c^2}\delta(c'-c)\delta(\mu'-\mu). \tag{3.10c}$$

A solution of a discrete-ordinates version of Eq. (3.5) can be written as [16]

$$\psi_0(\tau, c, \pm\mu_i) = \pm A_1c\mu_i + A_2(c^2 - 5/2) + B_1 + B_2[(c^2-5/2)\tau \mp \mu_i A(c)] + \boldsymbol{P}(c)\sum_{j=3}^J\left[A_j\boldsymbol{\Phi}(\nu_j, \pm\mu_i)\mathrm{e}^{-\tau/\nu_j} + B_j\boldsymbol{\Phi}(\nu_j, \mp\mu_i)\mathrm{e}^{\tau/\nu_j}\right], \tag{3.11}$$

for $i = 1, 2, \ldots, N$, where N is the order of the half-range quadrature with nodes $\{\mu_i\}$ and weights $\{w_i\}$ used to approximate integrals over [0, 1]. On the right side of Eq. (3.11), the first four terms are exact solutions (evaluated at the quadrature points) of Eq. (3.5), one of which is expressed in terms of the Chapman–Enskog function $A(c)$ (see definition and calculation methods in Refs. [27,28]). The last term is the ADO solution, which is expressed in terms of the separation constants $\{\nu_j\}$, the eigenvectors $\{\boldsymbol{\Phi}(\nu_j, \pm\mu_i)\}$, and the row vector

$$\boldsymbol{P}(c) = \left[P_0(2\mathrm{e}^{-c} - 1) \quad P_1(2\mathrm{e}^{-c} - 1) \cdots P_K(2\mathrm{e}^{-c} - 1)\right], \tag{3.12}$$

whose $K + 1$ components are the basis functions used to approximate the c-dependence of the solution. As discussed in detail in Ref. [16], the separation constants and the eigenvectors can be found from the solution of an eigensystem of order $J = N(K + 1)$. It should be noted that the separation constants for $j = 1$ and $j = 2$ approach unbounded values as the order of the approximation is increased and, for this reason, the corresponding solutions have been excluded from the summation and were replaced by the exact solutions. Once the coefficients A_j and B_j, $j = 1, 2, \ldots, J$, are determined, the solution expressed by Eq. (3.11) becomes completely known.

For the temperature-jump problem, the temperature perturbation [16,18]

$$T(\tau) = \frac{4}{3\pi^{1/2}} \int_0^\infty \int_{-1}^1 \mathrm{e}^{-c^2} \psi_0(\tau, c, \mu)(c^2 - 3/2)c^2 \,\mathrm{d}\mu \,\mathrm{d}c \tag{3.13}$$

must satisfy the Welander condition [29]

$$\lim_{\tau\to\infty} T'(\tau) = \kappa, \tag{3.14}$$

where $T'(\tau)$ denotes the derivative of $T(\tau)$ and κ is considered to be specified. By substituting Eq. (3.11) into Eq. (3.13) and applying Eq. (3.14), we conclude that $B_2 = \kappa$ and $B_j = 0$ for $j = 3, 4, \ldots, J$. In addition, we find that the temperature perturbation can be written, after the normalization $\kappa = 1$ is used, as

$$T(\tau) = T_{\text{asy}}(\tau) + \sum_{j=3}^{J} A_j T_j \mathrm{e}^{-\tau/\nu_j}, \tag{3.15}$$

where the asymptotic component is

$$T_{\text{asy}}(\tau) = A_2 + \tau \tag{3.16}$$

and

$$T_j = \frac{4}{3\pi^{1/2}} [\boldsymbol{P}_2 - (3/2)\boldsymbol{P}_0] \sum_{n=1}^{N} w_n [\boldsymbol{\Phi}(\nu_j, \mu_n) + \boldsymbol{\Phi}(\nu_j, -\mu_n)], \tag{3.17}$$

with

$$\boldsymbol{P}_n = \int_0^\infty \mathrm{e}^{-c^2} \boldsymbol{P}(c) c^{n+2} \,\mathrm{d}c. \tag{3.18}$$

When Eq. (3.11) is substituted into Eq. (3.6) evaluated at the quadrature nodes $\{\mu_i\}$ and the resulting equations are multiplied by $\boldsymbol{P}^T(c)\mathrm{e}^{-c^2}c^2$ and integrated over c from 0 to ∞, we obtain a system of J linear algebraic equations for the coefficients A_j, $j = 1, 2, \ldots, J$. Once this system is solved, we find the desired temperature-jump coefficient ζ from

$$\zeta \equiv [T_{\text{asy}}(0)/T'_{\text{asy}}(0)] = A_2. \tag{3.19}$$

4. The viscous-slip problem

In the case of the viscous-slip (or Kramers') problem, $h(z, \boldsymbol{c})$ is also taken to be a perturbation from an absolute Maxwellian distribution, and so the velocity distribution function can be expressed as in Eq. (3.1). However, since in this case there is a flow in the x-direction (parallel to the bounding surface), we write [25]

$$h(\tau/\varepsilon_0, \boldsymbol{c}) = \psi_1(\tau, c, \mu)(1 - \mu^2)^{1/2} \cos\phi, \tag{4.1}$$

where $\psi_1(\tau, c, \mu)$ is to be determined. Using Eq. (4.1) in Eq. (2.1) with $S(\boldsymbol{c}) = 0$ and in Eq. (2.6), multiplying the resulting equations by $\cos\phi$ and integrating both over ϕ, we can effectively reduce Kramers' problem with the CL boundary condition to solving

$$c\mu \frac{\partial}{\partial\tau} \psi_1(\tau, c, \mu) + \nu(c)\psi_1(\tau, c, \mu) = \int_0^\infty \int_{-1}^1 \mathrm{e}^{-c'^2} f(\mu', \mu) \times \mathcal{P}_1(c', \mu' : c, \mu)\psi_1(\tau, c', \mu')c'^2 \,\mathrm{d}\mu' \,\mathrm{d}c' \tag{4.2}$$

for $\tau \in (0, \infty)$, $c \in [0, \infty)$, and $\mu \in [-1, 1]$, subject to

$$\psi_1(0, c, \mu) = \int_0^\infty \int_0^1 \psi_1(0, c', -\mu')R_1(c', -\mu' : c, \mu)c'^2 \,\mathrm{d}\mu' \,\mathrm{d}c' \tag{4.3}$$

for $c \in [0, \infty)$ and $\mu \in (0, 1]$. In Eq. (4.2),

$$f(\mu', \mu) = \left(\frac{1 - \mu'^2}{1 - \mu^2}\right)^{1/2} \tag{4.4}$$

and the kernel $\mathcal{P}_1(c', \mu' : c, \mu)$ can be written as

$$\mathcal{P}_1(c', \mu' : c, \mu) = (1/2) \sum_{n=1}^{\infty} (2n + 1) P_n^1(\mu') P_n^1(\mu) \mathcal{P}_n(c', c), \tag{4.5}$$

where $P_n^1(x)$ is used to denote the normalized associated Legendre function

$$P_n^m(x) = \left[\frac{(n - m)!}{(n + m)!}\right]^{1/2} (1 - x^2)^{m/2} \frac{\mathrm{d}^m}{\mathrm{d}x^m} P_n(x) \tag{4.6}$$

for $m = 1$. In Eq. (4.3), $R_1(c', -\mu' : c, \mu)$ is given by [18]

$$R_1(c', -\mu' : c, \mu) = \frac{4c'\mu'}{\widehat{\alpha}\alpha_n} \mathrm{sgn}(1 - \alpha_t) S(c', -\mu' : c, \mu) \times U_1(c', -\mu' : c, \mu). \tag{4.7}$$

Equation (4.7) is valid for all of the allowed values of $\widehat{\alpha}$ and α_n, except the following special cases:

$$\lim_{\widehat{\alpha}\to 0} R_1(c', -\mu' : c, \mu) = \frac{2\mu'}{\alpha_n r(\mu')} \mathrm{sgn}(1 - \alpha_t) \times S(c', -\mu' : c, \mu)\delta[c'r(\mu') - cr(\mu)], \tag{4.8a}$$

$$\lim_{\alpha_n\to 0} R_1(c', -\mu' : c, \mu) = \frac{2}{\widehat{\alpha}} \mathrm{sgn}(1 - \alpha_t) \times U_1(c', -\mu' : c, \mu)\delta(c'\mu' - c\mu), \tag{4.8b}$$

and

$$\lim_{\widehat{\alpha}\to 0} \lim_{\alpha_n\to 0} R_1(c', -\mu' : c, \mu) = \frac{1}{c^2} \mathrm{sgn}(1 - \alpha_t)\delta(c' - c)\delta(\mu' - \mu). \tag{4.8c}$$

As discussed in detail in Ref. [17], a solution of a discrete-ordinates version of Eq. (4.2) is

$$\psi_1(\tau, c, \pm\mu_i) = A_1 c + B_1[c\tau \mp \mu_i B(c)] + \boldsymbol{P}(c) \sum_{j=2}^{J} \left[A_j \boldsymbol{\Phi}(\nu_j, \pm\mu_i)\mathrm{e}^{-\tau/\nu_j} + B_j \boldsymbol{\Phi}(\nu_j, \mp\mu_i)\mathrm{e}^{\tau/\nu_j}\right], \tag{4.9}$$

for $i = 1, 2, \ldots, N$. As before, N is the order of the half-range quadrature used to approximate integrals defined over [0, 1]. On the right side of Eq. (4.9), the first two terms are exact solutions (evaluated at the quadrature points) of Eq. (4.2), one of which includes the Chapman–Enskog function $B(c)$ (see definition and calculation methods in Refs. [27,28]), and the last term is the ADO solution. The coefficients A_j and B_j, $j = 1, 2, \ldots, N$, are to be determined. At this point, it is important to note that, even though we are using the same notation as in Section 3 to denote the separation constants $\{\nu_j\}$ and the eigenvectors $\{\boldsymbol{\Phi}(\nu_j, \pm\mu_i)\}$ in Eq. (4.9), these quantities are found in this section by solving a different eigensystem than the one that is relevant to Section 3. See Refs. [17,25] for details. In this case, only one of the separation constants approaches an unbounded value as the order of the approximation is increased and, for this reason, the corresponding solution (for $j = 1$) has been replaced by the exact solutions in Eq. (4.9).

For Kramers' problem, the bulk velocity [17,18]

$$u(\tau) = \frac{1}{\pi^{1/2}} \int_0^\infty \int_{-1}^1 \mathrm{e}^{-c^2} \psi_1(\tau, c, \mu) c^3 (1-\mu^2)\, \mathrm{d}\mu\, \mathrm{d}c, \tag{4.10}$$

must satisfy

$$\lim_{\tau\to\infty} u'(\tau) = \kappa_P, \tag{4.11}$$

where $u'(\tau)$ denotes the derivative of $u(\tau)$ and κ_P is considered to be specified. By substituting Eq. (4.9) into Eq. (4.10) and applying Eq. (4.11), we conclude that $B_1 = 2\kappa_P$ and $B_j = 0$ for $j = 2, 3, \ldots, J$. In addition, we find that the bulk velocity can be written, after the normalization $\kappa_P = 1$ is used, as

$$u(\tau) = u_{\text{asy}}(\tau) + \sum_{j=2}^{J} A_j N_j \mathrm{e}^{-\tau/\nu_j}, \tag{4.12}$$

where the asymptotic component is

$$u_{\text{asy}}(\tau) = (A_1/2) + \tau \tag{4.13}$$

and

$$N_j = \frac{1}{\pi^{1/2}} \boldsymbol{P}_1 \sum_{n=1}^{N} w_n (1-\mu_n^2)[\boldsymbol{\Phi}(\nu_j, \mu_n) + \boldsymbol{\Phi}(\nu_j, -\mu_n)]. \tag{4.14}$$

When Eq. (4.9) is substituted into Eq. (4.3) evaluated at the quadrature nodes $\{\mu_i\}$ and the resulting equations are multiplied by $\boldsymbol{P}^T(c)\mathrm{e}^{-c^2}c^2$ and integrated over c from 0 to ∞, we obtain a system of J linear algebraic equations for the coefficients A_j, $j = 1, 2, \ldots, J$. Once this system is solved, we find the desired viscous-slip coefficient ζ_P from

$$\zeta_P \equiv [u_{\text{asy}}(0)/u'_{\text{asy}}(0)] = A_1/2. \tag{4.15}$$

5. The thermal-creep problem

As mentioned in Section 2, $h(z, \boldsymbol{c})$ is taken to represent a perturbation from a local Maxwellian distribution in the case of the thermal-creep problem, and so the velocity distribution function is now given as [17,18]

$$f(x, z, \boldsymbol{v}) = f_0(v)[1 + (c^2 - 5/2)\kappa_T x + h(z, \boldsymbol{c})], \tag{5.1}$$

where κ_T is the constant gradient of the temperature along the x-direction (parallel to the bounding surface). Since $h(z, \boldsymbol{c})$ can be expressed for this problem as in Eq. (4.1), the governing equation for the thermal-creep problem is similar to Eq. (4.2), but with a source term [17,18]

$$S_T(c) = c(c^2 - 5/2)\kappa_T/\varepsilon_0 \tag{5.2}$$

added to the left side. The CL boundary condition to be satisfied in this case is the same as the one given by Eq. (4.3).

A solution for this problem that does not diverge as $\tau \to \infty$ is given by [17]

$$\psi_1(\tau, c, \pm\mu_i) = -(\kappa_T/\varepsilon_0)A(c) + A_1 c + \boldsymbol{P}(c) \sum_{j=2}^{J} A_j \boldsymbol{\Phi}(\nu_j, \pm\mu_i) \mathrm{e}^{-\tau/\nu_j}, \tag{5.3}$$

where the first term is a particular solution, the last term is the ADO solution used in Section 4, and the middle term is an exact solution of the homogeneous equation, which, as before, replaces the term that corresponds to an unbounded separation constant in the summation. By substituting Eq. (5.3) into a discrete-ordinates version of Eq. (4.3) and applying the same projection technique used in Section 4 on the resulting equations, we obtain a linear system of order J for the coefficients A_j, $j = 1, 2, \ldots, J$. Once this system is solved, we find the thermal-slip coefficient ζ_T from

$$\zeta_T \equiv u_{\text{asy}}(0) = A_1/2. \tag{5.4}$$

We note that, in order to generate numerical results for the thermal-slip coefficient, we have considered the normalization $\kappa_T = \varepsilon_0$ in this work.

6. Numerical results

Before reporting our numerical results for the slip and jump coefficients, we would like to mention that we were able to simplify some of the elements of the matrices of coefficients and the right-hand sides of the linear systems that have to be solved for the three problems. The fact that the double integrals of the CL kernels that are listed next could be evaluated analytically was very helpful for that purpose. For the temperature-jump problem, we have used

$$\int_0^\infty \int_0^1 R_0(c', -\mu' : c, \mu)\, \mathrm{d}\mu' c'^2\, \mathrm{d}c' = 1, \tag{6.1a}$$

$$\int_0^\infty \int_0^1 R_0(c', -\mu' : c, \mu)\mu'\, \mathrm{d}\mu' c'^3\, \mathrm{d}c' = \begin{cases} c\mu, & \alpha_n = 0, \\ (\pi\alpha_n)^{1/2}\mathrm{e}^{-\xi}\,[(\xi + 1/2)I_0(\xi) + \xi I_1(\xi)], & \alpha_n \neq 0, \end{cases} \tag{6.1b}$$

$$\int_0^\infty \int_0^1 R_0(c', -\mu' : c, \mu)\, \mathrm{d}\mu' c'^2 (c'^2 - 5/2)\, \mathrm{d}c' = (c^2 - 5/2) + \alpha_n f(c, \mu) + \widehat{\alpha} g(c, \mu), \tag{6.1c}$$

and

$$\int_0^\infty \int_0^1 R_0(c', -\mu' : c, \mu)\mu'\, \mathrm{d}\mu' c'^3 (c'^2 - 5/2)\, \mathrm{d}c' = \begin{cases} c\mu[(c^2 - 5/2) + \widehat{\alpha} g(c, \mu)], & \alpha_n = 0, \\ (\pi\alpha_n)^{1/2}\mathrm{e}^{-\xi}\,[p_0(c, \mu)I_0(\xi) + p_1(c, \mu)I_1(\xi)], & \alpha_n \neq 0, \end{cases} \tag{6.1d}$$

where we define

$$\xi \Rightarrow \xi(c, \mu) = (1 - \alpha_n)c^2\mu^2/(2\alpha_n), \tag{6.2}$$

$$f(c, \mu) = 1 - c^2\mu^2, \tag{6.3a}$$

$$g(c, \mu) = 1 - c^2(1 - \mu^2), \tag{6.3b}$$

$$p_0(c, \mu) = (\xi + 1/2)\{(c^2 - 5/2) + \alpha_n[f(c, \mu) + 1/2] + \widehat{\alpha} g(c, \mu)\} + \alpha_n \xi/2, \tag{6.4a}$$

and

$$p_1(c, \mu) = \xi\{(c^2 - 5/2) + \alpha_n[f(c, \mu) + 1] + \widehat{\alpha} g(c, \mu)\}. \tag{6.4b}$$

For the slip problems, along with the result

$$\int_0^\infty \int_0^1 f(\mu', \mu) R_1(c', -\mu' : c, \mu)\, \mathrm{d}\mu' c'^3\, \mathrm{d}c' = (1 - \alpha_t)c, \tag{6.5a}$$

which is given (in a different notation) by Eq. (6.1) of Ref. [25], we have used

$$\int_0^\infty \int_0^1 f(\mu', \mu) R_1(c', -\mu' : c, \mu)\mu'\, \mathrm{d}\mu' c'^4\, \mathrm{d}c' = \begin{cases} (1 - \alpha_t)c^2\mu, & \alpha_n = 0, \\ (\pi\alpha_n)^{1/2}(1 - \alpha_t)c\mathrm{e}^{-\xi}\,[(\xi + 1/2)I_0(\xi) + \xi I_1(\xi)], & \alpha_n \neq 0, \end{cases} \tag{6.5b}$$

Table 1
The viscous-slip coefficient ζ_P for the Cercignani–Lampis boundary condition.

α_t	α_n	BGK	S	GJ	MRS	CES	LBE
0.25	0.0	6.44457	6.45236	9.69217	2.84519	2.86938	2.87030
0.25	0.25	6.41210	6.41796	9.63716	2.83156	2.85713	2.85783
0.25	0.5	6.38380	6.38817	9.58981	2.81961	2.84723	2.84788
0.25	0.75	6.35783	6.36102	9.54698	2.80857	2.83865	2.83930
0.25	1.0	6.33355	6.33579	9.50748	2.79819	2.83104	2.83167
0.5	0.0	2.86119	2.86657	4.30879	1.26236	1.26388	1.26451
0.5	0.25	2.84035	2.84469	4.27426	1.25355	1.25609	1.25659
0.5	0.5	2.82188	2.82539	4.24388	1.24572	1.24966	1.25013
0.5	0.75	2.80477	2.80757	4.21595	1.23843	1.24401	1.24448
0.5	1.0	2.78865	2.79084	4.18985	1.23154	1.23893	1.23941
0.75	0.0	1.64258	1.64614	2.47498	7.24491(−1)	7.20388(−1)	7.20885(−1)
0.75	0.25	1.63254	1.63568	2.45863	7.20218(−1)	7.16670(−1)	7.17108(−1)
0.75	0.5	1.62350	1.62629	2.44396	7.16367(−1)	7.13543(−1)	7.13964(−1)
0.75	0.75	1.61505	1.61751	2.43027	7.12755(−1)	7.10761(−1)	7.11180(−1)
0.75	1.0	1.60701	1.60919	2.41730	7.09316(−1)	7.08231(−1)	7.08655(−1)
1.0	0.5	1.01619	1.01837	1.53106	4.48208(−1)	4.42921(−1)	4.43338(−1)
1.25	0.0	6.27438(−1)	6.28616(−1)	9.44797(−1)	2.76806(−1)	2.72169(−1)	2.72487(−1)
1.25	0.25	6.36792(−1)	6.38206(−1)	9.59554(−1)	2.80842(−1)	2.75568(−1)	2.75938(−1)
1.25	0.5	6.45455(−1)	6.47113(−1)	9.73309(−1)	2.84571(−1)	2.78526(−1)	2.78925(−1)
1.25	0.75	6.53722(−1)	6.55635(−1)	9.86508(−1)	2.88122(−1)	2.81223(−1)	2.81633(−1)
1.25	1.0	6.61701(−1)	6.63879(−1)	9.99315(−1)	2.91543(−1)	2.83729(−1)	2.84138(−1)
1.5	0.0	3.58078(−1)	3.58578(−1)	5.38646(−1)	1.58039(−1)	1.54873(−1)	1.55059(−1)
1.5	0.25	3.76152(−1)	3.76974(−1)	5.66734(−1)	1.65890(−1)	1.61402(−1)	1.61678(−1)
1.5	0.5	3.93123(−1)	3.94330(−1)	5.93375(−1)	1.73229(−1)	1.67175(−1)	1.67509(−1)
1.5	0.75	4.09472(−1)	4.11127(−1)	6.19288(−1)	1.80272(−1)	1.72501(−1)	1.72861(−1)
1.5	1.0	4.25373(−1)	4.27538(−1)	6.44735(−1)	1.87094(−1)	1.77502(−1)	1.77855(−1)
1.75	0.0	1.57395(−1)	1.57513(−1)	2.36446(−1)	6.95074(−2)	6.80329(−2)	6.80894(−2)
1.75	0.25	1.83607(−1)	1.84004(−1)	2.76591(−1)	8.09664(−2)	7.74846(−2)	7.76359(−2)
1.75	0.5	2.08549(−1)	2.09376(−1)	3.15293(−1)	9.18018(−2)	8.59743(−2)	8.61923(−2)
1.75	0.75	2.32799(−1)	2.34202(−1)	3.53421(−1)	1.02273(−1)	9.38957(−2)	9.41343(−2)
1.75	1.0	2.56568(−1)	2.58694(−1)	3.91296(−1)	1.12476(−1)	1.01410(−1)	1.01613(−1)
2.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2.0	0.25	3.38159(−2)	3.39479(−2)	5.10981(−2)	1.48791(−2)	1.22413(−2)	1.22739(−2)
2.0	0.5	6.64069(−2)	6.69256(−2)	1.01102(−1)	2.90985(−2)	2.34025(−2)	2.34593(−2)
2.0	0.75	9.83830(−2)	9.95418(−2)	1.50944(−1)	4.29356(−2)	3.39306(−2)	3.39569(−2)
2.0	1.0	1.29964(−1)	1.32020(−1)	2.00983(−1)	5.64914(−2)	4.40148(−2)	4.39317(−2)

and

$$\int_0^\infty \int_0^1 f(\mu',\mu)R_1(c',-\mu':c,\mu)\,\mathrm{d}\mu' c'^3(c'^2-5/2)\,\mathrm{d}c' = (1-\alpha_t)c\left\{(c^2-5/2)+\alpha_n f(c,\mu)+\widehat{\alpha}[1+g(c,\mu)]\right\}. \quad (6.5c)$$

Except for the use of high-order expansions for the rigid-sphere scattering kernel, as mentioned in the Introduction, and the use of the results expressed by Eqs. (6.1)–(6.5), the implementation of our solutions follows Ref. [18], and so we believe we do not need to discuss the numerical aspects of our solutions in detail here.

In Tables 1 and 2, we report our converged numerical results for the viscous-slip and thermal-slip coefficients for several values of the CL accommodation coefficients α_t and α_n. In addition to the LBE results, we list in these tables numerical results for the five kinetic models considered in this work: the BGK [30], S [31], GJ [32], MRS [33], and CES [34] models, which were implemented as discussed in detail in Ref. [33]. We note that the results listed in these tables for $\alpha_t = 1.0$ and $\alpha_n = 0.5$ are also valid for any other value of $\alpha_n \in [0, 1]$, when $\alpha_t = 1.0$. In Table 3, we list similar results for the temperature-jump coefficient. Since the CL kernel $R_0(c', -\mu' : c, \mu)$ that is expressed by Eq. (3.8) can easily be seen to be symmetrical about $\alpha_t = 1.0$, it is sufficient to consider values of $\alpha_t \in [0, 1]$ in a tabulation of the temperature-jump coefficient, as done in this work.

We note that our results for the models were obtained simply by using the special forms of the Chapman–Enskog functions $A(c)$ and $B(c)$ appropriate to each model [33] in our general (LBE) formulation and by noting that, in the case of the models, the infinite-order expansions of Eqs. (3.7) and (4.5) reduce to finite-order expansions, with upper limit $L = 1$ or 2 in the summations, depending on the model [33]. On the other hand, in the case of the LBE, the infinite-order kernels of Eqs. (3.7) and (4.5) were truncated arbitrarily at L and the value of L was varied between 10 and 150 to check the convergence of the numerical results. The coefficients $\{\mathcal{P}^{(n)}(c', c)\}$ in Eqs. (3.7) and (4.5) are given explicitly for the models in Ref. [33] and can be computed for the LBE with rigid-sphere scattering as discussed in Appendix A of Ref. [24].

The accuracy of the numerical results that can be obtained from a computational implementation of our solutions depends on the specific choices of four approximation parameters: the order K of the basis function expansion used to approximate the c-dependence of the solution, the order N of the half-range Gaussian quadrature used to approximate integrals defined over $[0, 1]$ in the ADO method, the order M of the Gaussian quadrature used to evaluate integrals over $[0, \infty]$, and, in the cases of the LBE and the CES model, the number K_s of splines used to compute [27] the Chapman–Enskog functions $A(c)$ and $B(c)$. The numerical results of Tables 1–3 are thought to be accurate to within ±1 in the last reported figure, and were obtained by observing numerical convergence as K was varied between 10 and 35, N was varied between 40 and 300, M was varied between 100 and 400, and $K_s - 2$ was varied between 320 and 1280. While a substantial computational effort (typically hours of CPU time per case on a

Table 2
The thermal-slip coefficient ζ_T for the Cercignani–Lampis boundary condition.

α_t	α_n	BGK	S	GJ	MRS	CES	LBE
0.25	0.0	2.85062(−1)	4.39613(−1)	6.80068(−1)	1.84369(−1)	1.87923(−1)	1.83227(−1)
0.25	0.25	3.10347(−1)	4.76987(−1)	7.35122(−1)	2.01333(−1)	2.01484(−1)	1.97426(−1)
0.25	0.5	3.35083(−1)	5.13417(−1)	7.88520(−1)	2.17970(−1)	2.14338(−1)	2.10966(−1)
0.25	0.75	3.59336(−1)	5.48967(−1)	8.40318(−1)	2.34345(−1)	2.26572(−1)	2.23939(−1)
0.25	1.0	3.83161(−1)	5.83717(−1)	8.90649(−1)	2.50496(−1)	2.38260(−1)	2.36426(−1)
0.5	0.0	3.18891(−1)	4.93891(−1)	7.66267(−1)	2.05154(−1)	2.02219(−1)	1.96448(−1)
0.5	0.25	3.35189(−1)	5.17326(−1)	7.99729(−1)	2.16359(−1)	2.10547(−1)	2.05393(−1)
0.5	0.5	3.51345(−1)	5.40585(−1)	8.32955(−1)	2.27443(−1)	2.18615(−1)	2.14104(−1)
0.5	0.75	3.67333(−1)	5.63580(−1)	8.65749(−1)	2.38420(−1)	2.26411(−1)	2.22562(−1)
0.5	1.0	3.83161(−1)	5.86302(−1)	8.98082(−1)	2.49302(−1)	2.33954(−1)	2.30788(−1)
0.75	0.0	3.51566(−1)	5.42280(−1)	8.37299(−1)	2.26937(−1)	2.17180(−1)	2.12088(−1)
0.75	0.25	3.59453(−1)	5.53449(−1)	8.52986(−1)	2.32435(−1)	2.21103(−1)	2.16351(−1)
0.75	0.5	3.67369(−1)	5.64713(−1)	8.68869(−1)	2.37925(−1)	2.24977(−1)	2.20580(−1)
0.75	0.75	3.75275(−1)	5.75981(−1)	8.84783(−1)	2.43398(−1)	2.28772(−1)	2.24738(−1)
0.75	1.0	3.83161(−1)	5.87231(−1)	9.00680(−1)	2.48852(−1)	2.32487(−1)	2.28821(−1)
1.0	0.5	3.83161(−1)	5.87362(−1)	9.01046(−1)	2.48787(−1)	2.32283(−1)	2.28546(−1)
1.25	0.0	4.13743(−1)	6.30546(−1)	9.61422(−1)	2.70138(−1)	2.46623(−1)	2.44289(−1)
1.25	0.25	4.06336(−1)	6.20244(−1)	9.47235(−1)	2.64889(−1)	2.43057(−1)	2.40377(−1)
1.25	0.5	3.98727(−1)	6.09557(−1)	9.32378(−1)	2.59549(−1)	2.39413(−1)	2.36361(−1)
1.25	0.75	3.90993(−1)	5.98624(−1)	9.17076(−1)	2.54156(−1)	2.35748(−1)	2.32318(−1)
1.25	1.0	3.83161(−1)	5.87494(−1)	9.01410(−1)	2.48722(−1)	2.32080(−1)	2.28271(−1)
1.5	0.0	4.43374(−1)	6.72335(−1)	1.01969	2.90822(−1)	2.58859(−1)	2.57683(−1)
1.5	0.25	4.28997(−1)	6.52619(−1)	9.92942(−1)	2.80499(−1)	2.52101(−1)	2.50193(−1)
1.5	0.5	4.14071(−1)	6.31890(−1)	9.64467(−1)	2.69915(−1)	2.45074(−1)	2.42374(−1)
1.5	0.75	3.98771(−1)	6.10449(−1)	9.34735(−1)	2.59158(−1)	2.37916(−1)	2.34409(−1)
1.5	1.0	3.83161(−1)	5.88406(−1)	9.03918(−1)	2.48263(−1)	2.30673(−1)	2.26361(−1)
1.75	0.0	4.72109(−1)	7.12450(−1)	1.07499	3.11054(−1)	2.67445(−1)	2.67086(−1)
1.75	0.25	4.51165(−1)	6.84410(−1)	1.03784	2.95649(−1)	2.57964(−1)	2.56375(−1)
1.75	0.5	4.29198(−1)	6.54530(−1)	9.97634(−1)	2.79752(−1)	2.47933(−1)	2.44998(−1)
1.75	0.75	4.06495(−1)	6.23276(−1)	9.55071(−1)	2.63513(−1)	2.37584(−1)	2.33268(−1)
1.75	1.0	3.83161(−1)	5.90821(−1)	9.10397(−1)	2.46988(−1)	2.26996(−1)	2.21298(−1)
2.0	0.0	5.00000(−1)	7.50000(−1)	1.12500	3.31456(−1)	2.71128(−1)	2.71128(−1)
2.0	0.25	4.72856(−1)	7.15015(−1)	1.08022	3.10638(−1)	2.59505(−1)	2.57606(−1)
2.0	0.5	4.44115(−1)	6.77221(−1)	1.03094	2.89072(−1)	2.46986(−1)	2.42973(−1)
2.0	0.75	4.14166(−1)	6.37235(−1)	9.78028(−1)	2.66954(−1)	2.33906(−1)	2.27690(−1)
2.0	1.0	3.83161(−1)	5.95284(−1)	9.21783(−1)	2.44365(−1)	2.20382(−1)	2.11931(−1)

Table 3
The temperature-jump coefficient ζ for the Cercignani–Lampis boundary condition.

α_t	α_n	BGK	S	GJ	MRS	CES	LBE
0.0	0.25	1.65478(1)	2.48217(1)	3.75568(1)	1.08307(1)	1.13002(1)	1.12585(1)
0.0	0.5	7.63078	1.14462(1)	1.74875(1)	4.92173	5.25355	5.20913
0.0	0.75	4.64213	6.96320	1.07616(1)	2.94068	3.23223	3.18680
0.0	1.0	3.14720	4.72080	7.40012	1.94808	2.22257	2.17680
0.25	0.0	1.00184(1)	1.50276(1)	2.31934(1)	6.37080	6.96782	6.90033
0.25	0.25	5.78950	8.68425	1.31047(1)	3.80300	3.89342	3.89548
0.25	0.5	3.84176	5.76263	8.64609	2.54583	2.55575	2.56267
0.25	0.75	2.72408	4.08612	6.13702	1.80181	1.80822	1.81158
0.25	1.0	2.00553	3.00830	4.54361	1.31364	1.33370	1.33270
0.5	0.0	5.76952	8.65428	1.36213(1)	3.55999	4.08447	4.01199
0.5	0.25	3.88593	5.82889	8.92213	2.49479	2.64263	2.62986
0.5	0.5	2.78041	4.17061	6.29960	1.82230	1.85234	1.85420
0.5	0.75	2.05839	3.08759	4.63631	1.36218	1.35515	1.35994
0.5	1.0	1.55658	2.33487	3.50324	1.03131	1.01630	1.02047
0.75	0.0	4.57103	6.85655	1.09177(1)	2.76885	3.27280	3.19758
0.75	0.25	3.22226	4.83339	7.47197	2.03476	2.21131	2.19146
0.75	0.5	2.36605	3.54907	5.39723	1.53272	1.58383	1.58151
0.75	0.75	1.77973	2.66959	4.02259	1.17067	1.17140	1.17468
0.75	1.0	1.35977	2.03966	3.06071	9.00829(−1)	8.82367(−1)	8.86916(−1)
1.0	0.0	4.27007	6.40510	1.02382(1)	2.57043	3.06924	2.99316
1.0	0.25	3.04475	4.56712	7.08466	1.91145	2.09643	2.07450
1.0	0.5	2.25090	3.37635	5.14774	1.45156	1.50985	1.50614
1.0	0.75	1.70032	2.55048	3.84919	1.11526	1.11968	1.12231
1.0	1.0	1.30272	1.95407	2.93392	8.62189(−1)	8.44123(−1)	8.48578(−1)

Table 4
The viscous-slip coefficient ζ_P for the Maxwell boundary condition.

α	BGK	S	GJ	MRS	CES	LBE
0.1	1.710313(1)	1.711289(1)	2.568772(1)	7.555154	7.653509	7.654919
0.2	8.224902	8.233445	1.236595(1)	3.631871	3.668384	3.669668
0.3	5.255112	5.262546	7.907298	2.319749	2.336314	2.337472
0.4	3.762619	3.769046	5.665008	1.660482	1.667589	1.668626
0.5	2.861190	2.866704	4.309695	1.262406	1.264266	1.265184
0.6	2.255410	2.260100	3.398205	9.949673(−1)	9.936912(−1)	9.944960(−1)
0.7	1.818667	1.822617	2.740594	8.022106(−1)	7.990129(−1)	7.997103(−1)
0.8	1.487654	1.490942	2.241873	6.561584(−1)	6.518023(−1)	6.523989(−1)
0.9	1.227198	1.229898	1.849257	5.412667(−1)	5.362646(−1)	5.367673(−1)
1.0	1.016191	1.018372	1.531065	4.482081(−1)	4.429214(−1)	4.433376(−1)

Table 5
The thermal-slip coefficient ζ_T for the Maxwell boundary condition.

α	BGK	S	GJ	MRS	CES	LBE
0.1	2.641783(−1)	3.990953(−1)	6.039153(−1)	1.741220(−1)	1.815785(−1)	1.806217(−1)
0.2	2.781510(−1)	4.224826(−1)	6.433307(−1)	1.824896(−1)	1.882732(−1)	1.865245(−1)
0.3	2.919238(−1)	4.451916(−1)	6.808852(−1)	1.908334(−1)	1.946586(−1)	1.922669(−1)
0.4	3.055019(−1)	4.672507(−1)	7.167056(−1)	1.991559(−1)	2.007561(−1)	1.978564(−1)
0.5	3.188906(−1)	4.886867(−1)	7.509074(−1)	2.074599(−1)	2.065853(−1)	2.032999(−1)
0.6	3.320949(−1)	5.095248(−1)	7.835963(−1)	2.157476(−1)	2.121642(−1)	2.086041(−1)
0.7	3.451195(−1)	5.297892(−1)	8.148692(−1)	2.240217(−1)	2.175091(−1)	2.137752(−1)
0.8	3.579692(−1)	5.495027(−1)	8.448150(−1)	2.322847(−1)	2.226350(−1)	2.188189(−1)
0.9	3.706483(−1)	5.686868(−1)	8.735154(−1)	2.405388(−1)	2.275555(−1)	2.237407(−1)
1.0	3.831612(−1)	5.873623(−1)	9.010457(−1)	2.487868(−1)	2.322833(−1)	2.285458(−1)

Table 6
The temperature-jump coefficient ζ for the Maxwell boundary condition.

α	BGK	S	GJ	MRS	CES	LBE
0.1	2.145012(1)	3.217519(1)	4.827543(1)	1.421449(1)	1.449039(1)	1.450952(1)
0.2	1.034747(1)	1.552120(1)	2.329286(1)	6.854927	6.950372	6.967199
0.3	6.630514	9.945770	1.492827(1)	4.391407	4.429768	4.444478
0.4	4.760333	7.140499	1.071906(1)	3.152106	3.164151	3.176926
0.5	3.629125	5.443688	8.172655	2.402649	2.400633	2.411643
0.6	2.867615	4.301423	6.458189	1.898241	1.888245	1.897653
0.7	2.317534	3.476301	5.219548	1.533963	1.519428	1.527387
0.8	1.899741	2.849612	4.278658	1.257352	1.240397	1.247053
0.9	1.570264	2.355396	3.536575	1.039258	1.021273	1.026763
1.0	1.302716	1.954073	2.933920	8.621892(−1)	8.441228(−1)	8.485784(−1)

personal computer) is required to generate numerical results as accurate as those of Tables 1–3, we have observed that the CPU time needed to obtain results accurate enough for practical use is much less. For example, we have been able to generate numerical results good to at least three significant figures for all of the LBE cases listed in Tables 1–3 in less than six minutes of CPU time on an Intel Core 2 Duo machine running at 2.6 GHz. Additional results for the Maxwell boundary condition, which is characterized by a single accommodation coefficient α, are given in Tables 4–6 and are also thought to be accurate to within ± 1 in the last reported figure. As expected, the Maxwell results for the case of purely diffuse reflection ($\alpha = 1.0$) are in complete agreement with the CL results for the cases $\alpha_t = 1.0$ of the slip coefficients and for the case $\alpha_t = 1.0$ and $\alpha_n = 1.0$ of the temperature-jump coefficient.

It can be seen from the results reported in Tables 1–6 that the CES model gives the best results when compared to the LBE. However, this model is somewhat more challenging to implement than the others, since it requires a numerical evaluation of the Chapman–Enskog functions $A(c)$ and $B(c)$. Among the simpler models, MRS is the only that gives results which are consistent with the LBE results. Nevertheless, the other models (BGK, S, and GJ) can look much better when the desired quantities are expressed in terms of conveniently defined, alternative mean-free path units. To show this, we report in Tables 7–12 numerical results expressed in terms of viscosity units for the viscous-slip coefficient and in terms of thermal-conductivity units for the thermal-slip and temperature-jump coefficients. We note that the results expressed in terms of viscosity units in Tables 7 and 10 can be obtained simply by dividing the corresponding results of Tables 1 and 4 by ε_p, the numerical values of which are given for the LBE and each of the models in Ref. [33]. Similarly, the results expressed in terms of thermal-conductivity units in Tables 8, 9, 11 and 12 can be obtained by dividing the corresponding results of Tables 2, 3, 5 and 6 by ε_t (see also Ref. [33] for numerical values of this parameter for the LBE and the models). In the cases of the slip coefficients for both the CL and the Maxwell boundary conditions and the temperature-jump coefficient for the Maxwell boundary condition (Tables 7, 8 and 10–12), we can see that the CES model still performs better than the other models (with the exception of one entry in Table 11) and the MRS model is the second best. However, in the case of the temperature-jump coefficient for the CL boundary condition (Table 9) this situation changes. While the CES model still performs better than the others for 17 of the 24 cases studied, the BGK and S models are the best for the remaining 7 cases, which include the four cases for which $\alpha_t = 0.0$. When the CES model is excluded from the comparison, the MRS model is the best for 13 cases and the BGK and S models are the best for 11 cases.

An additional reason to report our numerical results in different mean-free path units is the fact that all of the previous works with which we wish to compare our results make use of viscosity and/or thermal-conductivity units. Thus, repeating our results in the same units as in those works facilitates such comparisons. As discussed

Table 7
The viscous-slip coefficient ζ_P/ε_p for the Cercignani–Lampis boundary condition.

α_t	α_n	BGK	S	GJ	MRS	CES	LBE
0.25	0.0	6.44457	6.45236	6.46145	6.43792	6.39020	6.39226
0.25	0.25	6.41210	6.41796	6.42477	6.40709	6.36293	6.36448
0.25	0.5	6.38380	6.38817	6.39321	6.38004	6.34087	6.34233
0.25	0.75	6.35783	6.36102	6.36465	6.35507	6.32178	6.32321
0.25	1.0	6.33355	6.33579	6.33832	6.33159	6.30481	6.30622
0.5	0.0	2.86119	2.86657	2.87253	2.85639	2.81470	2.81610
0.5	0.25	2.84035	2.84469	2.84950	2.83647	2.79735	2.79846
0.5	0.5	2.82188	2.82539	2.82925	2.81874	2.78303	2.78408
0.5	0.75	2.80477	2.80757	2.81063	2.80225	2.77046	2.77151
0.5	1.0	2.78865	2.79084	2.79323	2.78665	2.75915	2.76021
0.75	0.0	1.64258	1.64614	1.64999	1.63934	1.60433	1.60543
0.75	0.25	1.63254	1.63568	1.63909	1.62967	1.59605	1.59702
0.75	0.5	1.62350	1.62629	1.62930	1.62095	1.58908	1.59002
0.75	0.75	1.61505	1.61751	1.62018	1.61278	1.58289	1.58382
0.75	1.0	1.60701	1.60919	1.61154	1.60500	1.57725	1.57820
1.0	0.5	1.01619	1.01837	1.02071	1.01418	9.86401(−1)	9.87328(−1)
1.25	0.0	6.27438(−1)	6.28616(−1)	6.29865(−1)	6.26341(−1)	6.06129(−1)	6.06837(−1)
1.25	0.25	6.36792(−1)	6.38206(−1)	6.39703(−1)	6.35473(−1)	6.13699(−1)	6.14524(−1)
1.25	0.5	6.45455(−1)	6.47113(−1)	6.48872(−1)	6.43911(−1)	6.20287(−1)	6.21174(−1)
1.25	0.75	6.53722(−1)	6.55635(−1)	6.57672(−1)	6.51946(−1)	6.26294(−1)	6.27207(−1)
1.25	1.0	6.61701(−1)	6.63879(−1)	6.66210(−1)	6.59687(−1)	6.31875(−1)	6.32784(−1)
1.5	0.0	3.58078(−1)	3.58578(−1)	3.59097(−1)	3.57602(−1)	3.44908(−1)	3.45321(−1)
1.5	0.25	3.76152(−1)	3.76974(−1)	3.77823(−1)	3.75365(−1)	3.59447(−1)	3.60063(−1)
1.5	0.5	3.93123(−1)	3.94330(−1)	3.95584(−1)	3.91973(−1)	3.72304(−1)	3.73049(−1)
1.5	0.75	4.09472(−1)	4.11127(−1)	4.12859(−1)	4.07908(−1)	3.84165(−1)	3.84967(−1)
1.5	1.0	4.25373(−1)	4.27538(−1)	4.29823(−1)	4.23346(−1)	3.95303(−1)	3.96088(−1)
1.75	0.0	1.57395(−1)	1.57513(−1)	1.57631(−1)	1.57277(−1)	1.51512(−1)	1.51637(−1)
1.75	0.25	1.83607(−1)	1.84004(−1)	1.84394(−1)	1.83206(−1)	1.72561(−1)	1.72898(−1)
1.75	0.5	2.08549(−1)	2.09376(−1)	2.10196(−1)	2.07724(−1)	1.91468(−1)	1.91953(−1)
1.75	0.75	2.32799(−1)	2.34202(−1)	2.35614(−1)	2.31418(−1)	2.09109(−1)	2.09640(−1)
1.75	1.0	2.56568(−1)	2.58694(−1)	2.60864(−1)	2.54504(−1)	2.25844(−1)	2.26297(−1)
2.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2.0	0.25	3.38159(−2)	3.39479(−2)	3.40654(−2)	3.36676(−2)	2.72619(−2)	2.73344(−2)
2.0	0.5	6.64069(−2)	6.69256(−2)	6.74013(−2)	6.58424(−2)	5.21183(−2)	5.22446(−2)
2.0	0.75	9.83830(−2)	9.95418(−2)	1.00630(−1)	9.71521(−2)	7.55646(−2)	7.56232(−2)
2.0	1.0	1.29964(−1)	1.32020(−1)	1.33989(−1)	1.27825(−1)	9.80224(−2)	9.78373(−2)

next, comparisons were done only with numerical results from works where the CL boundary condition is used in the calculation of the slip and/or jump coefficients (i.e., no work based solely on the Maxwell boundary condition was considered for this purpose).

With regard to the numerical results with five figures of accuracy that are given in Ref. [18] for the LBE subject to the CL boundary condition, we have found that the slip coefficients listed in Tables II and III of that work [18] display a maximum deviation of one unit in the last (fifth) figure, while the jump coefficient listed in Table IV displays a maximum deviation of two units in the last figure, which occurs for some of the special cases ($\alpha_t = 0.0$ or $\alpha_n = 0.0$) considered in that work. The six-figure results reported for the LBE with the Maxwell boundary condition in Table I of Ref. [18] have a maximum deviation of one unit in the last (sixth) figure.

While Ref. [18] is, to the best of our knowledge, the only published work that provides numerical results based on the LBE and the CL boundary condition for the slip and jump coefficients, there are some additional works based on model equations and the CL boundary condition. Concerning the seven-figure results for the slip coefficients that were obtained from the BGK and S model equations with the CL boundary condition in Ref. [20], we have found that they agree perfectly with our results, when rounded to six significant figures. The slip-coefficient results with seven figures of accuracy that are reported in that work [20] for the BGK, S, and CES models with the Maxwell boundary condition are in perfect agreement with the results of our work. Numerical results for the slip and jump coefficients using the S model and the CL boundary condition were also reported in Ref. [21], but the tabulations provide only four (sometimes three) significant figures. We have confirmed that these results are correct with a maximum deviation of four units in the fourth figure. Finally, with regard to the numerical results for the temperature-jump coefficient that were obtained from the BGK and S model equations and the CL boundary condition and are reported with six figures of accuracy in Tables 2–4 of Ref. [22], we have found that they agree very well with the results of our method (maximum difference of one unit in the sixth significant figure).

7. Concluding remarks

Accurate numerical results are reported in this work for the viscous-slip, thermal-slip, and temperature-jump coefficients calculated with the linearized Boltzmann equation for rigid-sphere interactions (and five kinetic model equations) subject to either the Cercignani–Lampis or the Maxwell boundary condition. The CES model was found to be the one that yields the numerical results that are closest to the LBE results, except in the limit $\alpha_t \to 0$ of the temperature-jump coefficient for the CL boundary condition, where the BGK and S models give better results (when expressed in convenient mean-free path units). Among the simple models considered in this work (BGK, S, GJ, and MRS), the MRS model yields numerical results that agree better with the LBE results than the other three (except in the above-mentioned limit of the

Table 8
The thermal-slip coefficient ζ_T/ε_t for the Cercignani–Lampis boundary condition.

α_t	α_n	BGK	S	GJ	MRS	CES	LBE
0.25	0.0	2.85062(−1)	2.93076(−1)	3.02252(−1)	2.78120(−1)	2.76508(−1)	2.69598(−1)
0.25	0.25	3.10347(−1)	3.17991(−1)	3.26721(−1)	3.03710(−1)	2.96461(−1)	2.90491(−1)
0.25	0.5	3.35083(−1)	3.42278(−1)	3.50453(−1)	3.28806(−1)	3.15375(−1)	3.10413(−1)
0.25	0.75	3.59336(−1)	3.65978(−1)	3.73475(−1)	3.53508(−1)	3.33375(−1)	3.29501(−1)
0.25	1.0	3.83161(−1)	3.89144(−1)	3.95844(−1)	3.77872(−1)	3.50574(−1)	3.47874(−1)
0.5	0.0	3.18891(−1)	3.29261(−1)	3.40563(−1)	3.09474(−1)	2.97543(−1)	2.89052(−1)
0.5	0.25	3.35189(−1)	3.44884(−1)	3.55435(−1)	3.26376(−1)	3.09797(−1)	3.02213(−1)
0.5	0.5	3.51345(−1)	3.60390(−1)	3.70202(−1)	3.43097(−1)	3.21667(−1)	3.15030(−1)
0.5	0.75	3.67333(−1)	3.75720(−1)	3.84777(−1)	3.59656(−1)	3.33138(−1)	3.27475(−1)
0.5	1.0	3.83161(−1)	3.90868(−1)	3.99147(−1)	3.76071(−1)	3.44238(−1)	3.39578(−1)
0.75	0.0	3.51566(−1)	3.61520(−1)	3.72133(−1)	3.42333(−1)	3.19556(−1)	3.12065(−1)
0.75	0.25	3.59453(−1)	3.68966(−1)	3.79105(−1)	3.50627(−1)	3.25329(−1)	3.18337(−1)
0.75	0.5	3.67369(−1)	3.76475(−1)	3.86164(−1)	3.58909(−1)	3.31028(−1)	3.24559(−1)
0.75	0.75	3.75275(−1)	3.83988(−1)	3.93237(−1)	3.67164(−1)	3.36613(−1)	3.30677(−1)
0.75	1.0	3.83161(−1)	3.91487(−1)	4.00302(−1)	3.75391(−1)	3.42079(−1)	3.36685(−1)
1.0	0.5	3.83161(−1)	3.91575(−1)	4.00465(−1)	3.75293(−1)	3.41779(−1)	3.36280(−1)
1.25	0.0	4.13743(−1)	4.20364(−1)	4.27299(−1)	4.07501(−1)	3.62878(−1)	3.59444(−1)
1.25	0.25	4.06336(−1)	4.13496(−1)	4.20993(−1)	3.99583(−1)	3.57632(−1)	3.53688(−1)
1.25	0.5	3.98727(−1)	4.06371(−1)	4.14390(−1)	3.91529(−1)	3.52270(−1)	3.47779(−1)
1.25	0.75	3.90993(−1)	3.99083(−1)	4.07589(−1)	3.83393(−1)	3.46877(−1)	3.41830(−1)
1.25	1.0	3.83161(−1)	3.91662(−1)	4.00627(−1)	3.75195(−1)	3.41480(−1)	3.35876(−1)
1.5	0.0	4.43374(−1)	4.48223(−1)	4.53197(−1)	4.38703(−1)	3.80882(−1)	3.79152(−1)
1.5	0.25	4.28997(−1)	4.35079(−1)	4.41307(−1)	4.23132(−1)	3.70938(−1)	3.68132(−1)
1.5	0.5	4.14071(−1)	4.21260(−1)	4.28652(−1)	4.07165(−1)	3.60599(−1)	3.56626(−1)
1.5	0.75	3.98771(−1)	4.06966(−1)	4.15438(−1)	3.90939(−1)	3.50067(−1)	3.44906(−1)
1.5	1.0	3.83161(−1)	3.92271(−1)	4.01741(−1)	3.74504(−1)	3.39410(−1)	3.33064(−1)
1.75	0.0	4.72109(−1)	4.74966(−1)	4.77772(−1)	4.69223(−1)	3.93515(−1)	3.92987(−1)
1.75	0.25	4.51165(−1)	4.56273(−1)	4.61261(−1)	4.45984(−1)	3.79565(−1)	3.77227(−1)
1.75	0.5	4.29198(−1)	4.36353(−1)	4.43393(−1)	4.22005(−1)	3.64805(−1)	3.60488(−1)
1.75	0.75	4.06495(−1)	4.15518(−1)	4.24476(−1)	3.97508(−1)	3.49579(−1)	3.43228(−1)
1.75	1.0	3.83161(−1)	3.93881(−1)	4.04621(−1)	3.72580(−1)	3.34000(−1)	3.25616(−1)
2.0	0.0	5.00000(−1)	5.00000(−1)	5.00000(−1)	5.00000(−1)	3.98935(−1)	3.98935(−1)
2.0	0.25	4.72856(−1)	4.76677(−1)	4.80099(−1)	4.68596(−1)	3.81832(−1)	3.79039(−1)
2.0	0.5	4.44115(−1)	4.51481(−1)	4.58196(−1)	4.36063(−1)	3.63413(−1)	3.57508(−1)
2.0	0.75	4.14166(−1)	4.24824(−1)	4.34679(−1)	4.02698(−1)	3.44167(−1)	3.35020(−1)
2.0	1.0	3.83161(−1)	3.96856(−1)	4.09681(−1)	3.68623(−1)	3.24267(−1)	3.11833(−1)

Table 9
The temperature-jump coefficient ζ/ε_t for the Cercignani–Lampis boundary condition.

α_t	α_n	BGK	S	GJ	MRS	CES	LBE
0.0	0.25	1.65478(1)	1.65478(1)	1.66919(1)	1.63380(1)	1.66270(1)	1.65657(1)
0.0	0.5	7.63078	7.63078	7.77223	7.42441	7.73001	7.66466
0.0	0.75	4.64213	4.64213	4.78291	4.43600	4.75586	4.68902
0.0	1.0	3.14720	3.14720	3.28894	2.93866	3.27027	3.20293
0.25	0.0	1.00184(1)	1.00184(1)	1.03082(1)	9.61032	1.02524(1)	1.01531(1)
0.25	0.25	5.78950	5.78950	5.82432	5.73681	5.72873	5.73177
0.25	0.5	3.84176	3.84176	3.84271	3.84037	3.76051	3.77069
0.25	0.75	2.72408	2.72408	2.72757	2.71803	2.66059	2.66554
0.25	1.0	2.00553	2.00553	2.01938	1.98161	1.96239	1.96092
0.5	0.0	5.76952	5.76952	6.05389	5.37023	6.00985	5.90319
0.5	0.25	3.88593	3.88593	3.96539	3.76337	3.88834	3.86955
0.5	0.5	2.78041	2.78041	2.79982	2.74893	2.72552	2.72824
0.5	0.75	2.05839	2.05839	2.06058	2.05483	1.99396	2.00099
0.5	1.0	1.55658	1.55658	1.55700	1.55573	1.49537	1.50151
0.75	0.0	4.57103	4.57103	4.85229	4.17680	4.81556	4.70488
0.75	0.25	3.22226	3.22226	3.32088	3.06942	3.25370	3.22449
0.75	0.5	2.36605	2.36605	2.39877	2.31210	2.33043	2.32701
0.75	0.75	1.77973	1.77973	1.78782	1.76594	1.72359	1.72841
0.75	1.0	1.35977	1.35977	1.36032	1.35890	1.29831	1.30500
1.0	0.0	4.27007	4.27007	4.55031	3.87749	4.51605	4.40410
1.0	0.25	3.04475	3.04475	3.14874	2.88341	3.08467	3.05239
1.0	0.5	2.25090	2.25090	2.28788	2.18967	2.22158	2.21612
1.0	0.75	1.70032	1.70032	1.71075	1.68237	1.64749	1.65135
1.0	1.0	1.30272	1.30272	1.30396	1.30061	1.24203	1.24859

Table 10
The viscous-slip coefficient ζ_P/ε_p for the Maxwell boundary condition.

α	BGK	S	GJ	MRS	CES	LBE
0.1	1.710313(1)	1.711289(1)	1.712515(1)	1.709536(1)	1.704462(1)	1.704776(1)
0.2	8.224902	8.233445	8.243969	8.217986	8.169615	8.172474
0.3	5.255112	5.262546	5.271532	5.248993	5.203049	5.205629
0.4	3.762619	3.769046	3.776672	3.757241	3.713778	3.716085
0.5	2.861190	2.866704	2.873130	2.856500	2.815562	2.817607
0.6	2.255410	2.260100	2.265470	2.251354	2.212984	2.214776
0.7	1.818667	1.822617	1.827063	1.815195	1.779429	1.780982
0.8	1.487654	1.490942	1.494582	1.484717	1.451586	1.452914
0.9	1.227198	1.229898	1.232838	1.224747	1.194279	1.195399
1.0	1.016191	1.018372	1.020710	1.014179	9.864009(−1)	9.873277(−1)

Table 11
The thermal-slip coefficient ζ_T/ε_t for the Maxwell boundary condition.

α	BGK	S	GJ	MRS	CES	LBE
0.1	2.641783(−1)	2.660636(−1)	2.684068(−1)	2.626621(−1)	2.671726(−1)	2.657648(−1)
0.2	2.781510(−1)	2.816551(−1)	2.859247(−1)	2.752845(−1)	2.770231(−1)	2.744500(−1)
0.3	2.919238(−1)	2.967944(−1)	3.026157(−1)	2.878711(−1)	2.864184(−1)	2.828993(−1)
0.4	3.055019(−1)	3.115005(−1)	3.185358(−1)	3.004256(−1)	2.953902(−1)	2.911236(−1)
0.5	3.188906(−1)	3.257911(−1)	3.337366(−1)	3.129520(−1)	3.039673(−1)	2.991332(−1)
0.6	3.320949(−1)	3.396832(−1)	3.482650(−1)	3.254541(−1)	3.121761(−1)	3.069378(−1)
0.7	3.451195(−1)	3.531928(−1)	3.621641(−1)	3.379355(−1)	3.200405(−1)	3.145464(−1)
0.8	3.579692(−1)	3.663351(−1)	3.754733(−1)	3.504001(−1)	3.275826(−1)	3.219676(−1)
0.9	3.706483(−1)	3.791246(−1)	3.882291(−1)	3.628515(−1)	3.348226(−1)	3.292095(−1)
1.0	3.831612(−1)	3.915748(−1)	4.004647(−1)	3.752934(−1)	3.417790(−1)	3.362797(−1)

Table 12
The temperature-jump coefficient ζ/ε_t for the Maxwell boundary condition.

α	BGK	S	GJ	MRS	CES	LBE
0.1	2.145012(1)	2.145012(1)	2.145575(1)	2.144248(1)	2.132099(1)	2.134915(1)
0.2	1.034747(1)	1.034747(1)	1.035238(1)	1.034062(1)	1.022670(1)	1.025146(1)
0.3	6.630514	6.630514	6.634788	6.624413	6.517910	6.539555
0.4	4.760333	4.760333	4.764025	4.754934	4.655696	4.674493
0.5	3.629125	3.629125	3.632291	3.624383	3.532264	3.548464
0.6	2.867615	2.867615	2.870306	2.863486	2.778342	2.792185
0.7	2.317534	2.317534	2.319799	2.313975	2.235669	2.247380
0.8	1.899741	1.899741	1.901626	1.896708	1.825107	1.834900
0.9	1.570264	1.570264	1.571811	1.567715	1.502689	1.510768
1.0	1.302716	1.302716	1.303964	1.300608	1.242033	1.248589

temperature-jump coefficient) and is the only simple model that yields consistent results regardless of the choice of mean-free path.

Note added in revision: While revising our manuscript, we were informed of the recent death of Carlo Cercignani. It is with sadness that we take this opportunity to remember Carlo who almost single-handedly defined the field of rarefied gas dynamics. He will be greatly missed.

Acknowledgments

The authors are grateful to Luis C. Ogando Dacal and Onofre F. de Lima Neto from IEAv for computational resources that were used to generate part of the numerical results reported in this work.

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