THE ASTROPHYSICAL JOURNAL, Vol. 159, February 1970

MOMENTS OF CHANDRASEKHAR'S FUNCTIONS H_l AND H_r

ABSTRACT

The first twenty-two moments of Chandrasekhar's functions H_l and H_r are given.

Various moments of Chandrasekhar's H-functions (Chandrasekhar 1950) appear quite frequently in the solutions to problems of radiative transfer in finite and semi-infinite media, often in the definitions of fundamental quantities of interest. Although rigorous analytical expressions are generally available, it is sometimes more expedient for numerical calculations to represent H-functions by polynomial expansions, and thus the moments of H-functions may be so used to advantage. Further, we have found these moments useful for numerical verification of results obtained by the method of normal modes (Case 1960) for finite-slab and half-space applications.

As a matter of interest we have computed moments of the functions H_l and H_r related to the scattering of polarized light (Chandrasekhar 1950). The moments

$$a_{\beta,n} \triangleq \int_0^1 \mu^n H_\beta(\mu) d\mu\,, \quad \beta = l \quad \text{or} \quad r\,,$$

are displayed in Table 1 for $n = 0, 1, 2, \ldots, 21$.

TABLE 1

MOMENTS OF THE H_l- AND H_r-FUNCTIONS

n	$a_{l,n}$	$a_{r,n}$	n	$a_{l,n}$	$a_{r,n}$
0	2.29767688	1.19735662	11	0.27445235	0.10590702
1	1.34864649	0.61732831	12	0.25431038	0.09780290
2	0.96434354	0.41630804	13	0.23692548	0.09085080
3	0.75236170	0.31406290	14	0.22176734	0.08482138
4	0.61733813	0.25213343	15	0.20843357	0.07954241
5	0.52361722	0.21060099	16	0.19661329	0.07488199
6	0.45469730	0.18081401	17	0.18606250	0.07073743
7	0.40185783	0.15840770	18	0.17658699	0.06702757
8	0.36004671	0.14094151	19	0.16803029	0.06368744
9	0.32613139	0.12694401	20	0.16026487	0.06066438
10	0.29806486	0.11547534	21	0.15318577	0.05791530

Although these two H-functions had been tabulated previously (Chandrasekhar 1950), a more exhaustive compilation was utilized recently (Bond and Siewert 1967). Here, as in our earlier work, an 81-point improved Gaussian quadrature scheme (Kronrod 1965) was used for numerical integrations, and the computations were performed in double-precision arithmetic on an IBM 360/75. As an indication of the accuracy, the even moments were numerically verified to satisfy the identities

$$a_{\beta,0} = 1 + \tfrac{1}{2}k_\beta[(a_{\beta,0})^2 - (a_{\beta,1})^2]$$

and

$$a_{\beta,2n} = \frac{1}{2n+1} + \tfrac{1}{2}k_\beta\left[\sum_{r=0}^{2n-2} a_{\beta,2n-r}(a_{\beta,r} - a_{\beta,r+2})(-1)^r - a_{\beta,1}a_{\beta,2n-1} + a_{\beta,0}a_{\beta,2n}\right], \quad n \geq 1\,,$$

which are modifications of a form given by Busbridge (1960). Here $\beta = l$ or r, $k_l = \frac{3}{4}$, and $k_r = \frac{3}{8}$. The significant figures shown in the table are well within the accuracy indicated by the above verification.

This work was supported in part by the National Science Foundation through grant GK-3072.

G. R. Bond
C. E. Siewert

1969 June 25
Department of Nuclear Engineering
North Carolina State University

REFERENCES

Bond, G. R., and Siewert, C. E. 1967, *Ap. J.*, **150**, 357.
Busbridge, I. W. 1960, *The Mathematics of Radiative Transfer* (London: Cambridge University Press), § 12.
Case, K. M. 1960, *Ann. Phys.*, **9**, 1.
Chandrasekhar, S. 1950, *Radiative Transfer* (London: Oxford University Press).
Kronrod, A. S. 1965, *Nodes and Weights of Quadrature Formulas* (New York: Consultants Bureau).