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An Exact Analytical Solution of an Elementary Critical Condition

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ABSTRACT

A real solution of the critical condition for an unreflected nuclear reactor, described by age-diffusion theory, is reported.

The familiar critical condition, described by age-diffusion theory, for a bare nuclear reactor is

$$\frac{k\exp(-B^2\tau)}{1+B^2L^2}=1 \tag{1}$$

and is, of course, transcendental in the buckling B^2. Equation (1) can clearly be solved numerically by iteration; however, the equation can also be solved analytically and thus reduced to quadrature.

In a recent paper[1] exact analytical solutions for all of the (in general) complex solutions of

$$z\exp(z)=a \tag{2}$$

were reported, and since Eq. (1) can be cast in the form of Eq. (2), we wish simply to list here the final result for the desired real solution of Eq. (1):

$$B^2=\frac{1}{\tau}-\frac{1}{L^2}+\left[\frac{1}{L^2}-\frac{1}{\tau}+\frac{1}{\tau}\ln\left(\frac{k\tau}{L^2}\right)\right]\exp\left[\frac{1}{\pi}\int_0^1 f(x)\,\frac{dx}{1-x}\right] , \tag{3}$$

where

$$f(x)=\tan^{-1}\left[\frac{-\pi}{\frac{\tau}{L^2}+\ln\left(\frac{k\tau}{L^2}\right)+\frac{x}{1-x}-\ln\left(\frac{x}{1-x}\right)}\right] \tag{4}$$

is continuous and defined such that $f(0)=0$.

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[1] C. E. SIEWERT and E. E. BURNISTON, *J. Math. Anal. Appl.* (in press).