An Asymptotic Solution in the Theory of Neutron Moderation

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ABSTRACT

An asymptotic solution to the equation that describes the slowing down of neutrons is given.

ANALYSIS

The basic equation in the elementary theory of neutron moderation can be written as[1-3]

$$\Phi(u) = \delta(u) + c\int_{u-q}^{u} \Phi(u')\,K(u-u')du' \ , \qquad (1)$$

where

$\Phi(u)$ = flux

$\delta(u)$ = Dirac's delta functional

$c = \Sigma_s/\Sigma$

$q = -\ln\alpha$

$K(u) = \dfrac{e^{-u}}{1-\alpha},\ 0 \le u \le q$,

$K(u) = 0$, otherwise.

Following the work of Marshak,[1] Ferziger and Zweifel[2] and Williams[3] have expressed the asymptotic solution to Eq. (1) as

$$\Phi_{\text{asy}}(u) = \frac{c\{1-\exp[-q(s_0+1)]\}\exp(s_0u)}{1-\alpha-cq\exp[-q(s_0+1)]} \ , \qquad (2)$$

where s_0 is the appropriate real solution $\epsilon(-\infty, 0)$ of

$$(1-\alpha)(s+1) = c\{1-\exp[-q(s+1)]\} \ . \qquad (3)$$

Equation (3) has only two real solutions, one of which clearly is $s = -1$; we note, however, that $s_0 \neq -1$, except for the very special case for which $cq = 1-\alpha$.

In this Note we wish to report an exact expression for s_0, and thus we let $\beta = cq/(\alpha-1)$ and $\lambda = \beta + q(s+1)$ so that Eq. (3) can be written as

$$\lambda e^{\lambda} = \beta e^{\beta} \ . \qquad (4)$$

The work of Siewert and Burniston[4] can now be used to solve Eq. (4) for the real solution $\lambda \neq \beta$ and thus to establish an explicit expression for s_0:

$$s_0 = -1 - \frac{(\beta+1)}{q}\exp\left\{-\frac{1}{\pi}\int_0^{\infty}[f(x)-\pi]\frac{dx}{x-\beta}\right\} \ , \qquad (5)$$

where

$$f(x) = \tan^{-1}\left(\frac{\pi}{\beta+\ln|\beta|-x-\ln x}\right) \qquad (6)$$

is continuous and such that $f(0) = 0$. The solution given by Eq. (5) has been evaluated numerically to yield results accurate to five significant figures.

To report some numerical results, we write Eq. (5) as

$$s_0 = -1 - \frac{(\beta+1)}{q}K(\beta) \qquad (7)$$

and list in the accompanying table the function **$K(\beta)$**.

TABLE I

The Function $K(\beta)$

$-\beta$	$K(\beta)$	$-\beta$	$K(\beta)$	$-\beta$	$K(\beta)$
0.0	∞	0.400	2.6980	1.50	1.7484
0.005	7.3214	0.500	2.5129	2.00	1.5936
0.040	4.9944	0.600	2.3685	3.00	1.4107
0.080	4.2493	0.700	2.2516	5.00	1.2413
0.140	3.6745	0.800	2.1542	10.00	1.1111
0.200	3.3255	0.900	2.0715	50.00	1.0204
0.300	2.9494	0.999	2.0000	∞	1.0000

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[1]R. E. MARSHAK, *Rev. Mod. Phys.*, **19**, 185 (1947).

[2]J. H. FERZIGER and P. F. ZWEIFEL, *The Theory of Neutron Slowing Down in Nuclear Reactors*, Massachusetts Institute of Technology Press, Cambridge (1966).

[3]M. M. R. WILLIAMS, *The Slowing Down and Thermalization of Neutrons*, North-Holland Publishing Co., Amsterdam (1966).

[4]C. E. SIEWERT and E. E. BURNISTON, *J. Math. Anal. and Appl.*, **43**, 626 (1973).