Reprint, Technical Note, Nuclear Science & Engineering: 63, 95-96 (1977)

Technical Notes

On the Inverse Problem for Multigroup Neutron Transport Theory

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Received November 23, 1976
Revised December 20, 1976

ABSTRACT

The inverse problem for multigroup transport theory is solved for the cases of plane and spherical symmetry.

INTRODUCTION

In two recent papers,[1,2] the inverse problem based on the one-speed neutron transport equation was solved by two very different methods. Also, in regard to time-dependent one-group theory, two authors[3,4] have reported results from which the solution to the inverse problem can be deduced. Here we solve the inverse problem based on the multigroup model.

A traditional problem in neutron transport theory consists of specifying the physical parameters and boundary conditions and then seeking to establish the angular flux. Here for the inverse problem we intend to determine the scattering-fission law by expressing the transfer matrices in terms of the flux, which is presumed deducible from experimentation.

ANALYSIS

We consider an infinite subcritical[5] medium defined by

$$\mu \frac{\partial}{\partial x} \mathbf{\Psi}(x,\mu) + \mathbf{\Sigma}\mathbf{\Psi}(x,\mu) = \sum_{l=0}^{\infty} \left(\frac{2l+1}{2}\right) P_l(\mu)\mathbf{C}_l \times \int_{-1}^{1} \mathbf{\Psi}(x,\mu')P_l(\mu')d\mu' + \tfrac{1}{2}\,\delta(x)\mathbf{I} \quad . \tag{1}$$

Here $\mathbf{\Psi}(x,\mu)$ is an $n \times n$ matrix, the columns of which are the angular fluxes, $\mathbf{\Sigma}$ is the diagonal total cross-section matrix, and the elements of the transfer matrices $\mathbf{C}_l$ are the l'th angular components of the transfer cross sections for fission and scattering. From Eq. (1) it is clear that the α'th column of $\mathbf{\Psi}(x,\mu)$ is the angular flux vector corresponding to an inhomogeneous source only in the α'th group.

If we multiply Eq. (1) by $P_l(\mu)$ and integrate over μ, we find

$$(2l+1)\mathbf{\Delta}_l\mathbf{\Psi}_l(x) = \delta(x)\delta_{0,l}\mathbf{I} - (l+1)\mathbf{\Psi}'_{l+1}(x) - l\mathbf{\Psi}'_{l-1}(x) \quad , \tag{2}$$

[1] K. M. CASE, *Phys. Fluids*, **16**, 1607 (1973).
[2] N. J. McCORMICK and I. KUŠČER, *J. Math. Phys.*, **15**, 926 (1974).
[3] E. H. CANFIELD, *Nucl. Sci. Eng.*, **53**, 137 (1974).
[4] E. R. COHEN, *Nucl. Sci. Eng.*, **52**, 280 (1973).
[5] R. L. BOWDEN, *Trans. Theory Stat. Phys.*, **4**, 25 (1975).

where

$$\mathbf{\Delta}_l = \mathbf{\Sigma} - \mathbf{C}_l \tag{3}$$

and

$$\mathbf{\Psi}_l(x) = \int_{-1}^{1} \mathbf{\Psi}(x,\mu) P_l(\mu) d\mu \quad . \tag{4}$$

If we multiply Eq. (2), for $l = 0$, by $x^{2\alpha}$, $\alpha = 0, 1, 2, \ldots$, and integrate over x, we can write

$$\mathbf{\Delta}_0 \int_{-\infty}^{\infty} x^{2\alpha} \mathbf{\Psi}_0(x) dx = \delta_{\alpha,0} \mathbf{I} + (2\alpha) \int_{-\infty}^{\infty} x^{2\alpha-1} \mathbf{\Psi}_1(x) dx \quad , \tag{5}$$

which, for $\alpha = 0$, yields

$$\mathbf{\Delta}_0^{-1} = 2 \int_0^{\infty} \mathbf{\Psi}_0(x) dx \quad . \tag{6}$$

From Eq. (2) we see that

$$\mathbf{\Psi}_1(x) = -\tfrac{1}{3} \mathbf{\Delta}_1^{-1} \left[2\mathbf{\Psi}_2'(x) + \mathbf{\Psi}_0'(x) \right] \quad , \tag{7}$$

which can be used in Eq. (5) to obtain

$$\mathbf{\Delta}_0 \mathbf{M}_{2\alpha} - \tfrac{2}{3} \alpha(2\alpha - 1) \mathbf{\Delta}_1^{-1} \mathbf{M}_{2\alpha-2} = \tfrac{2}{3} \alpha(2\alpha - 1) \mathbf{\Delta}_1^{-1} \int_{-\infty}^{\infty} x^{2\alpha-2} \mathbf{\Psi}_2(x) dx \quad , \tag{8}$$

where the moments of the flux are denoted by

$$\mathbf{M}_\alpha = \int_0^{\infty} x^\alpha \mathbf{\Psi}_0(x) dx \quad . \tag{9}$$

We can solve Eq. (8) for $\alpha = 1$ to obtain

$$\mathbf{\Delta}_1^{-1} = 3\mathbf{\Delta}_0 \int_0^{\infty} x^2 \mathbf{\Psi}_0(x) dx \mathbf{\Delta}_0 \quad . \tag{10}$$

It is clear that we can continue the process of using Eq. (2) in Eq. (8) and integrating by parts to find all of the $\mathbf{\Delta}_l$ in terms of moments of the flux. To be specific, we list

$$\mathbf{\Delta}_0^{-1} = 2\mathbf{M}_0 \quad , \tag{11a}$$

$$\mathbf{\Delta}_1^{-1} = 3\mathbf{\Delta}_0 \mathbf{M}_2 \mathbf{\Delta}_0 \quad , \tag{11b}$$

$$\mathbf{\Delta}_2^{-1} = \tfrac{15}{16} \mathbf{\Delta}_1 \mathbf{\Delta}_0 \mathbf{M}_4 \mathbf{\Delta}_0 \mathbf{\Delta}_1 - \tfrac{5}{4} \mathbf{\Delta}_0^{-1} \quad , \tag{11c}$$

$$\begin{aligned} \mathbf{\Delta}_3^{-1} = {} & \tfrac{35}{288} \mathbf{\Delta}_2 \mathbf{\Delta}_1 \mathbf{\Delta}_0 \mathbf{M}_6 \mathbf{\Delta}_0 \mathbf{\Delta}_1 \mathbf{\Delta}_2 - \tfrac{175}{144} \mathbf{\Delta}_2 \mathbf{M}_4 \mathbf{\Delta}_0 \mathbf{\Delta}_1 \mathbf{\Delta}_2 \\ & - \tfrac{35}{9} \mathbf{\Delta}_1^{-1} \mathbf{M}_2 \mathbf{\Delta}_0 \mathbf{\Delta}_1 \mathbf{\Delta}_2 - \tfrac{28}{27} \mathbf{\Delta}_1^{-1} \quad , \end{aligned} \tag{11d}$$

and

$$\begin{aligned} \mathbf{\Delta}_4^{-1} = {} & \tfrac{35}{4096} \mathbf{\Delta}_3 \mathbf{\Delta}_2 \mathbf{\Delta}_1 \mathbf{\Delta}_0 \mathbf{M}_8 \mathbf{\Delta}_0 \mathbf{\Delta}_1 \mathbf{\Delta}_2 \mathbf{\Delta}_3 - \tfrac{245}{1536} \mathbf{\Delta}_3 \mathbf{\Delta}_2 \mathbf{M}_6 \mathbf{\Delta}_0 \mathbf{\Delta}_1 \mathbf{\Delta}_2 \mathbf{\Delta}_3 \\ & - \tfrac{245}{192} \mathbf{\Delta}_3 \mathbf{\Delta}_1^{-1} \mathbf{M}_4 \mathbf{\Delta}_0 \mathbf{\Delta}_1 \mathbf{\Delta}_2 \mathbf{\Delta}_3 \\ & - \tfrac{7}{48} (27\mathbf{\Delta}_1 + 28\mathbf{\Delta}_3) \mathbf{\Delta}_1^{-1} \mathbf{\Delta}_2^{-1} \mathbf{\Delta}_1^{-1} \mathbf{M}_2 \mathbf{\Delta}_0 \mathbf{\Delta}_1 \mathbf{\Delta}_2 \mathbf{\Delta}_3 \\ & - \tfrac{1}{720} (27\mathbf{\Delta}_1 + 28\mathbf{\Delta}_3) \mathbf{\Delta}_1^{-1} \mathbf{\Delta}_2^{-1} \mathbf{\Delta}_1^{-1} (27\mathbf{\Delta}_1 + 28\mathbf{\Delta}_3) \quad . \end{aligned} \tag{11e}$$

It is clear from Eqs. (11) that we can obtain the desired matrices $\mathbf{C}_l$ from the total cross sections, even moments of the flux matrices and the $\mathbf{C}_\alpha$ matrices for $\alpha = 0, 1, 2, \ldots, l - 1$.

Since we have the point-to-plane transformation,

$$\mathbf{\Psi}_{0,pt}(r) = -\frac{1}{2\pi r} \mathbf{\Psi}_{0,pl}'(r) \quad , \tag{12}$$

Eqs. (11) can readily be converted to the spherical case by noting that

$$\mathbf{M}_\alpha = \frac{2\pi}{\alpha + 1} \int_0^{\infty} r^{\alpha+2} \mathbf{\Psi}_{0,pt}(r) dr \quad . \tag{13}$$

ACKNOWLEDGMENT

The authors are grateful to N. J. McCormick for pointing out several references related to this work.