# On a possible experiment to evaluate the validity of the one-speed or constant cross section model of the neutron-transport equation

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The inverse problem for a half-space is solved (for isotropic scattering) to yield results that suggest an idealized experiment that could be used to evaluate in a new way the validity of the one-speed or constant cross section model of the neutron-transport equation.

## INTRODUCTION

Inverse problems in the theory of neutron diffusion have been discussed in recent years for finite[1,2] and infinite media.[3–5] Here we would like to investigate the half-space inverse problem for the one-speed or constant cross section model of the neutron-transport equation and to show how the established results suggest an experiment that could be used to evaluate the isotropic-scattering model of the neutron-transport equation.

## ANALYSIS

We consider the neutron-transport equation

$$\mu \frac{\partial}{\partial x} \psi(x,\mu) + \psi(x,\mu) = \frac{c}{2}\int_{-1}^{1} \psi(x,\mu')\, d\mu', \tag{1}$$

where $\psi(x,\mu)$ is the neutron angular flux, $x$ is the position variable measured in mean-free-paths, $\mu$ is the direction cosine, and

$$c = (\nu\Sigma_f + \Sigma_s)/\Sigma \tag{2}$$

is the mean number of secondary neutrons per collision. Traditionally for $c < 1$, we seek to solve Eq. (1) in a semi-infinite half-space such that

$$\psi(0,\mu) = F(\mu), \quad \mu > 0 \tag{3a}$$

and

$$\psi(\infty,\mu) = 0, \tag{3b}$$

where $F(\mu)$ is considered given. Here we consider that $F(\mu)$ is specified, that $\psi(0,-\mu)$, $\mu > 0$, can be measured experimentally, and that we wish to determine the mean number of secondaries $c$.

We know from the work of Chandrasekhar[6] that the exit flux can be computed from

$$\psi(0,-\mu) = \frac{c}{2} H(\mu) \int_0^1 H(x) F(x)\, x \frac{dx}{x+\mu}, \quad \mu > 0, \tag{4}$$

where $H(\mu)$ satisfies

$$H(\mu) = 1 + \frac{c}{2}\mu H(\mu) \int_0^1 H(x) \frac{dx}{x+\mu}. \tag{5}$$

It is clear that we cannot readily solve Eq. (4) for $c$ since $H(\mu)$ is a function of $c$. Moments of the exit distribution can be found by multiplying Eq. (4) by $\mu^\alpha$ and integrating over $\mu$. For example, after using Eq. (5), we can write

$$\psi_0 = \int_0^1 F(x)[H(x) - 1]\, dx, \tag{6a}$$

$$\psi_1 = \int_0^1 F(x)[-xH(x)\sqrt{1-c} + x]\, dx, \tag{6b}$$

and

$$\psi_2 = \int_0^1 F(x)\left[xH(x)\left(\frac{c}{2}H_1 + x\sqrt{1-c}\right) - x^2\right] dx, \tag{6c}$$

where

$$H_\alpha = \int_0^1 H(x)\, x^\alpha\, dx \tag{7a}$$

and

$$\psi_\alpha = \int_0^1 \psi(0,-\mu)\, \mu^\alpha\, d\mu. \tag{7b}$$

If we consider the special case of an isotropic incident flux, $F(\mu) = 1$, then the resulting version of Eq. (6a) yields

$$\psi_0^{(0)} = H_0 - 1 = (2/c)(1 - \sqrt{1-c}) - 1, \tag{8}$$

which can be solved for $c$ to yield

$$c = \frac{4\psi_0^{(0)}}{[\psi_0^{(0)} + 1]^2}. \tag{9}$$

Here we use

$$\psi_\alpha^{(\beta)} = \int_0^1 \psi^{(\beta)}(0,-\mu)\, \mu^\alpha\, d\mu, \tag{10}$$

where $\psi^{(\beta)}(x,\mu)$ denotes the solution of Eq. (1) corresponding to $F(\mu) = \mu^\beta$.

If we now consider $F(\mu) = \mu$, then Eqs. (6a) and (6b) can be used with the identity[6]

$$\sqrt{1-c}\, H_2 + (c/4) H_1^2 = \tfrac{1}{3} \tag{11}$$

to deduce

$$c = \frac{4\psi_1^{(1)}}{[\psi_0^{(1)} + \frac{1}{2}]^2}. \tag{12}$$

In a similar manner Eqs. (6a) and (6c) and the identity[7]

$$\sqrt{1-c}\, H_4 - (c/2)(\tfrac{1}{2}H_2^2 - H_3 H_1) = \tfrac{1}{5} \tag{13}$$

can be used to establish

$$c = \frac{4\psi_2^{(2)}}{[\psi_0^{(2)} + \frac{1}{3}]^2}. \tag{14}$$

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With the aid of Busbridge's identity[7] concerning moments of the $H$ function,

$$\sqrt{1-c}\,H_{2\alpha} + (c/4)(H_1H_{2\alpha-1} - H_2H_{2\alpha-2} + \cdots + H_{2\alpha-1}H_1) = \frac{1}{2\alpha+1}, \tag{15}$$

we can generalize Eqs. (9), (12), and (14) to obtain

$$c = \frac{4\psi_\beta^{(\beta)}}{[\psi_0^{(\beta)} + (\beta+1)^{-1}]^2}, \quad \beta = 0, 1, 2, 3, \cdots. \tag{16}$$

Generally when we apply Eq. (1) to physical problems we consider $c$ to be a constant and thus clearly not a function of the boundary conditions. It thus seems feasible that the manner in which $c$, as computed from Eq. (16) and the experimentally measured $\psi^{(\beta)}(0, -\mu)$, varies with $\beta$ would be a reasonable measure of the accuracy with which Eq. (1) represents the given physical problem. It also seems feasible that the multigroup version of Eq. (16) would offer a definition of the transfer cross sections alternative to the traditional one. The finite-slab inverse problem solved in Ref. 2 for the multigroup model could serve a similar purpose.

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