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Technical Notes

The Inverse Problem for a Finite Slab

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ABSTRACT

The finite-slab inverse problem for multigroup neutron transport theory is solved.

INTRODUCTION

In a recent Note,[1] the inverse problem for multigroup transport theory was discussed and solved for an infinite medium. Here we solve the inverse problem for the considerably more important case of a finite slab.[2] Traditionally, we seek to determine the angular flux after specifying the physical parameters of the medium and

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[1]C. E. SIEWERT, M. N. ÖZIŞIK, and Y. YENER, *Nucl. Sci. Eng.*, **63**, 95 (1977).

[2]S. PAHOR, *Phys. Rev.*, **175**, 218 (1968).

appropriate boundary conditions. For the inverse problem, we wish to determine the physical parameters from a measurement of the flux and the angular distribution of neutrons leaving the slab.

ANALYSIS

We consider a finite subcritical medium defined by

$$\mu \frac{\partial}{\partial x} \boldsymbol{\Psi}(x,\mu) + \boldsymbol{\Sigma}\boldsymbol{\Psi}(x,\mu) = \sum_{l=0}^{\infty}\left(\frac{2l+1}{2}\right) P_l(\mu)\mathbf{C}_l \int_{-1}^{1} \boldsymbol{\Psi}(x,\mu')P_l(\mu')d\mu' \ , \quad x \in [0,a] \ . \tag{1}$$

Here $\boldsymbol{\Psi}(x,\mu)$ is an $n \times n$ matrix, the columns of which are the angular fluxes, $\boldsymbol{\Sigma}$ is the diagonal total cross-section matrix, and the elements of the transfer matrices $\mathbf{C}_l$ are the l'th angular components of the transfer cross sections for fission and scattering. Since we allow $\boldsymbol{\Psi}(x,\mu)$ to be an $n \times n$ matrix, we consider boundary conditions of the form

$$\boldsymbol{\Psi}(0,\mu) = \mathbf{F}(\mu) \ , \qquad \mu > 0 \ , \tag{2a}$$

and

$$\boldsymbol{\Psi}(a,-\mu) = \mathbf{0} \ , \qquad \mu > 0 \ , \tag{2b}$$

where $\mathbf{F}(\mu)$ is an $n \times n$ matrix that we consider to be specified. We note that the α'th column of $\boldsymbol{\Psi}(x,\mu)$ is the angular flux vector corresponding to an incident distribution represented by the α'th column of $\mathbf{F}(\mu)$.

If we multiply Eq. (1) by $P_l(\mu)$ and integrate over μ, we find

$$(2l+1)\boldsymbol{\Delta}_l\boldsymbol{\Psi}_l(x) = -(l+1)\boldsymbol{\Psi}'_{l+1}(x) - l\boldsymbol{\Psi}'_{l-1}(x) \ , \tag{3}$$

where

$$\boldsymbol{\Delta}_l = \boldsymbol{\Sigma} - \mathbf{C}_l \tag{4}$$

and

$$\boldsymbol{\Psi}_l(x) = \int_{-1}^{1} P_l(\mu)\boldsymbol{\Psi}(x,\mu)d\mu \ . \tag{5}$$

If we multiply Eq. (3), for $l = 0$, by x^α, $\alpha = 0, 1, 2, 3, \ldots$, and integrate over x, we can write

$$\boldsymbol{\Delta}_0 \int_0^a x^\alpha \boldsymbol{\Psi}_0(x)\,dx = \delta_{\alpha,0}\boldsymbol{\Psi}_1(0) - a^\alpha\boldsymbol{\Psi}_1(a) + \alpha\int_0^a x^{\alpha-1}\boldsymbol{\Psi}_1(x)\,dx \ , \tag{6}$$

which, for $\alpha = 0$, yields

$$\boldsymbol{\Delta}_0^{-1} = \mathbf{M}_0[\boldsymbol{\Psi}_1(0) - \boldsymbol{\Psi}_1(a)]^{-1} \ , \tag{7}$$

where

$$\mathbf{M}_\alpha = \int_0^a x^\alpha \boldsymbol{\Psi}_0(x)\,dx \ . \tag{8}$$

From Eq. (3), we see that

$$\boldsymbol{\Psi}_1(x) = -\frac{1}{3}\boldsymbol{\Delta}_1^{-1}[2\boldsymbol{\Psi}'_2(x) + \boldsymbol{\Psi}'_0(x)] \ , \tag{9}$$

which can be used in Eq. (6) to obtain

$$\boldsymbol{\Delta}_0\mathbf{M}_\alpha - \frac{1}{3}\alpha(\alpha-1)\boldsymbol{\Delta}_1^{-1}\mathbf{M}_{\alpha-2} + a^\alpha\boldsymbol{\Psi}_1(a) - \frac{1}{3}\alpha\boldsymbol{\Delta}_1^{-1} \times \{\delta_{\alpha,1}[2\boldsymbol{\Psi}_2(0) + \boldsymbol{\Psi}_0(0)] - a^{\alpha-1}[2\boldsymbol{\Psi}_2(a) + \boldsymbol{\Psi}_0(a)]\} = \frac{2}{3}\alpha(\alpha-1)\boldsymbol{\Delta}_1^{-1}\int_0^a x^{\alpha-2}\boldsymbol{\Psi}_2(x)\,dx \ , \quad \alpha \geq 1 \ . \tag{10}$$

We can solve Eq. (10) for $\alpha = 1$ to obtain

$$\boldsymbol{\Delta}_1^{-1} = 3[\boldsymbol{\Delta}_0\mathbf{M}_1 + a\boldsymbol{\Psi}_1(a)]\{2[\boldsymbol{\Psi}_2(0) - \boldsymbol{\Psi}_2(a)] + \boldsymbol{\Psi}_0(0) - \boldsymbol{\Psi}_0(a)\}^{-1} \ . \tag{11}$$

It is clear that we can continue to use Eq. (3) in Eq. (10) and to integrate by parts to find all of the $\boldsymbol{\Delta}_l$ in terms of spatial moments of the flux $\mathbf{M}_\alpha$ and angular moments of the reflected and transmitted angular fluxes, $\boldsymbol{\Psi}_l(0)$ and $\boldsymbol{\Psi}_l(a)$. We list the following explicit results:

$$\boldsymbol{\Delta}_0^{-1} = \mathbf{M}_0\mathbf{N}_0^{-1} \ , \tag{12a}$$

$$\boldsymbol{\Delta}_1^{-1} = 3[\boldsymbol{\Delta}_0\mathbf{M}_1 + a\boldsymbol{\Psi}_1(a)]\mathbf{N}_1^{-1} \ , \tag{12b}$$

$$\boldsymbol{\Delta}_2^{-1} = \left\{\frac{15}{4}\boldsymbol{\Delta}_1\boldsymbol{\Delta}_0\mathbf{M}_2 - \frac{5}{2}\mathbf{M}_0 + \frac{15}{4}a^2\boldsymbol{\Delta}_1\boldsymbol{\Psi}_1(a) + \frac{5}{2}a[2\boldsymbol{\Psi}_2(a) + \boldsymbol{\Psi}_0(a)]\right\}\mathbf{N}_2^{-1} \ , \tag{12c}$$

$$\boldsymbol{\Delta}_3^{-1} = \left\{\frac{35}{12}\boldsymbol{\Delta}_2\boldsymbol{\Delta}_1\boldsymbol{\Delta}_0\mathbf{M}_3 - \frac{1}{6}[35\boldsymbol{\Delta}_2 + 28\boldsymbol{\Delta}_0]\mathbf{M}_1 + \frac{35}{12}a^3\boldsymbol{\Delta}_2\boldsymbol{\Delta}_1\boldsymbol{\Psi}_1(a) + \frac{35}{12}a^2\boldsymbol{\Delta}_2[2\boldsymbol{\Psi}_2(a) + \boldsymbol{\Psi}_0(a)] + 7a\boldsymbol{\Psi}_3(a)\right\}\mathbf{N}_3^{-1} \ , \tag{12d}$$

and

$$\boldsymbol{\Delta}_4^{-1} = \left\{\frac{105}{64}\boldsymbol{\Delta}_3\boldsymbol{\Delta}_2\boldsymbol{\Delta}_1\boldsymbol{\Delta}_0\mathbf{M}_4 - \frac{3}{16}[35\boldsymbol{\Delta}_3\boldsymbol{\Delta}_2 + 28\boldsymbol{\Delta}_3\boldsymbol{\Delta}_0 + 27\boldsymbol{\Delta}_1\boldsymbol{\Delta}_0]\mathbf{M}_2 + \frac{27}{8}\mathbf{M}_0 + \frac{105}{64}a^4\boldsymbol{\Delta}_3\boldsymbol{\Delta}_2\boldsymbol{\Delta}_1\boldsymbol{\Psi}_1(a) + \frac{35}{16}a^3\boldsymbol{\Delta}_3\boldsymbol{\Delta}_2[2\boldsymbol{\Psi}_2(a) + \boldsymbol{\Psi}_0(a)] + \frac{9}{16}a^2[14\boldsymbol{\Delta}_3\boldsymbol{\Psi}_3(a) - 9\boldsymbol{\Delta}_1\boldsymbol{\Psi}_1(a)] + \frac{9}{8}a[8\boldsymbol{\Psi}_4(a) - 3\boldsymbol{\Psi}_0(a)]\right\}\mathbf{N}_4^{-1} \ . \tag{12e}$$

In Eqs. (12), we have used

$$\mathbf{N}_l = (l+1)[\boldsymbol{\Psi}_{l+1}(0) - \boldsymbol{\Psi}_{l+1}(a)] + l[\boldsymbol{\Psi}_{l-1}(0) - \boldsymbol{\Psi}_{l-1}(a)] \ . \tag{13}$$

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